ISSN: 2366-1313

Love waves in the Newtonian viscous liquid layer in contact with transversely isotropic poroelastic layer and in-homogeneous half-space

C. Nageswaranath[1], V. Sravana Kumar [2], K. Elangovan[3]

[1]Department of Mathematics, BVRIT HYDERABAD College of Engineering for Women, Hyderabad - 500 090, Telangana

[2]Department of Mathematics, Sree Rama Engineering College, Tirupath-517507, Andhra Pradesh

[3]Mathematics Section, FEAT, Annamalai University, Chidambaram - 608002, Tamil Nādu

Abstract: A study on propagation of love waves in a viscous liquid layer bounded between initially stressed transversely isotropic poroelastic layer and in-homogeneous half-space is presented. The equations of motion in poroelastic layer have been formulated following Biot's theory. Solving the equations of motion the frequency equation for love waves is derived from which the phase velocity of waves has been studied. The effect of viscosity, ratio of thickness of the two layers on phase velocity have been observed. The frequency equations of few particular cases like viscous liquid layer in contact with in-homogeneous half-space and transversely isotropic poroelastic layer in contact with in-homogeneous half-space are derived. It is observed that phase velocity increases with an increase in viscosity coefficient.

Key words: Love waves, in-homogeneous, viscous liquid layer, transversely poroelastic, phase velocity, wave number, initially stressed.

1. Introduction

Deresiewicz (1962, 1962a) investigated the problem of propagation of Love waves in liquid- filled porous space. Bhangar (1978) studied propagation of Love waves in a system composed of a compressible viscous liquid layer sandwiched between homogeneous, elastic layer and homogeneous isotropic half-space. Biot(1956) developed and discussed a phenomenological theory of wave propagation in isotropic porous media. Chattopadhyay and De (1983) discussed propagation of Love waves at a rectangular irregular interface of a porous layer and isotropic elastic medium. Love waves propagating in an elastic layer overlying on poroelastic solid half-space is studied by Poonam Khurana and Vashistha (2001). The layer and half-space are considered under initial stress and interface between them is considered as loosely bonded. Gupta *et.al*. (2010) examined the effect of initial stress on Love waves in an anisotropic porous layer in contact with a pre-stressed, non-homogeneous elastic half-space. They concluded that as the porosity of the layer increases, the phase velocity of Love wave increases and an increase in compressive initial stresses in the porous layer decreases the velocity of Love waves. Propagation of waves in a Newtonian viscous liquid layer bounded by two poroelastic half-spaces is studied by Nageswaranath *et.al.* (2011). Possible bonding (welded interface, smooth interface and loosely bonded interface) for different values of the ratio of thickness of liquid layer to its shear viscosity is examined. Nageswaranath *et.al.* (2015) discussed the effect of viscous liquid layer on propagation of Love waves in viscous liquid layer bounded between transversely isotropic poroelastic layer and half-space. It is found that there is no influence of viscous liquid layer on phase velocity of Love waves when the liquid layer is bounded between poroelastic layer and half-space. Propagation of Love waves in an infinite poroelastic layer bounded between two compressible viscous liquids is examined by Nageswaranath *et.al.* (2016). Two poroelastic materials, sandstone saturated with water and sandstone saturated with kerosene have been considered for numerical study. It is noted that phase velocity increases with an increase in non-dimensional wavenumber. A closed-form solution for the propagation of Love waves in a transversely isotropic poroelastic layer bounded between two compressible viscous liquids is presented by Nageswarnath et. al.(2019). Vanitha sharma et.al. (2020) presented theoretical investigations of Love waves in a layered structure composed of a layer of finite thickness exhibiting heterogeneities in the form of void pores lying over a couple stress substrate which possesses inner microstructures. Bagno (2020) solved the problem of the propagation of acoustic waves in a layer of a compressible viscous fluid that interacts with an elastic half-space using the three dimensional linear equations of classical elasticity theory for the solid and the three-dimensional linearized Navier-Stokes equations for the compressible fluid.

In the present study, propagation of Love waves in a viscous liquid layer bounded by transversely isotropic poroelastic layer and in-homogeneous isotropic half-space is discussed. The equations of motion in a transversely isotropic poroelastic layer, liquid layer and in-homogeneous half-space are derived. The frequency equation for Love waves in a liquid layer bounded by transversely isotropic poroelastic layer and half-space is derived. Phase velocity of Love waves is computed and analyzed against non-dimensional wavenumber. The effects of viscosity of liquid is discussed and analyzed on the propagation of waves numerically. Also, the impact of initial stresses of both solids are observed. Few particular cases like viscous liquid layer in contact with in-homogeneous half-space and transversely isotropic poroelastic layer in contact with in-homogeneous half-space are obtained and frequency equations of both cases are derived. It is noticed that phase velocity increases as viscosity increases.

2. Formulation and Solution of the problem:

Let x, y, z be a rectangular co-ordinate system with x and y axes taken as horizontal and z-axis as positive downwards normal to the plane. Wave propagation is assumed to be two-dimensional (i.e. wave propagating only in xz- plane) and is taken in the direction of x-axis. It is also assumed that all variables of state are independent of y co-ordinate. Viscous liquid layer of thickness h_1 bounded between a transversely isotropic poroelastic layer of thickness h_2 and an in-homogeneous elastic half-space is considered. The poroelastic layer is assumed to be homogeneous and transversely isotropic with z-axis as axis of symmetry. The physical parameters with subscript (1) & (2) refer to solid layer and solid half-space, respectively. The parameters of viscous liquid layer are taken with no subscript. The boundaries of the geometry are taken as $z=0$, $z=h_1$ and $z=h_1+h_2$.

We examined Love waves on a transversely poroelastic layer with a thickness of h that was initially stressed and in contact with an initially stressed in-homogeneous half-space. The origin has been initiated at the interface of layer and half space. Wave motion is towards x-axis and the positive z-axis is taken towards the interior of the lower half space.

ISSN: 2366-1313

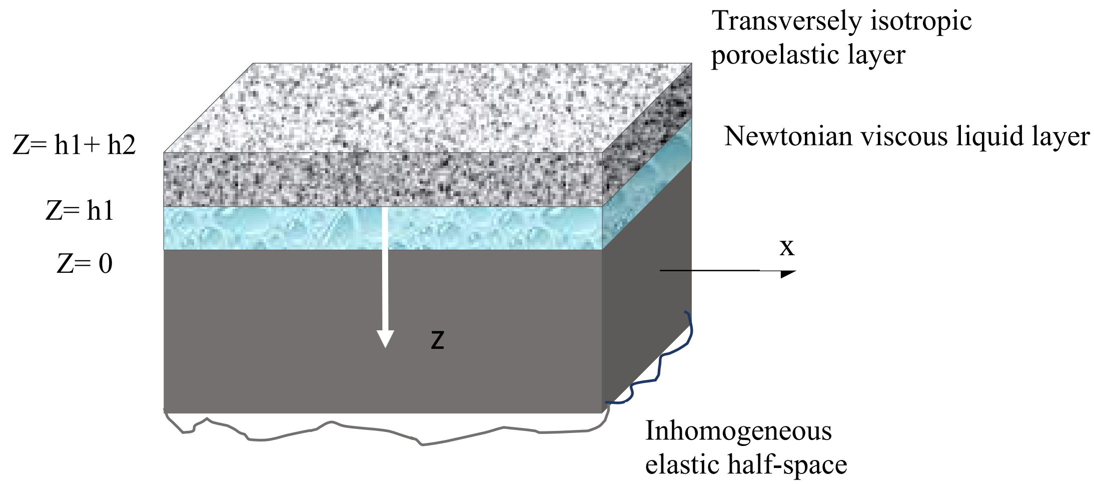

Figure. 1 Liquid layer bounded between poroelastic layer and Inhomogeneous half-space

2. 1. Solution in the poroelastic layer

In the absence of body forces and under initial stress *P*, the dynamic equations of motion (Biot, 1957) in a transversely isotropic poroelastic sheet are

$$\frac{\partial\sigma_{xx}}{\partial x}+\frac{\partial\sigma_{xy}}{\partial y}+\frac{\partial\sigma_{xz}}{\partial z}-P\left(\frac{\partial\omega_z}{\partial y}-\frac{\partial\omega_y}{\partial z}\right)=\frac{\partial^2}{\partial t^2}¿)$$

$$\frac{\partial\sigma_{xy}}{\partial x}+\frac{\partial\sigma_{yy}}{\partial y}+\frac{\partial\sigma_{yz}}{\partial z}-P\frac{\partial\omega_z}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{11}u_y+e_{12}U_y\right)$$

$$\frac{\partial\sigma_{zx}}{\partial x}+\frac{\partial\sigma_{zy}}{\partial y}+\frac{\partial\sigma_{zz}}{\partial z}-P\frac{\partial\omega_y}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{11}u_z+e_{12}U_z\right)$$

$$\frac{\partial s}{\partial x}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_x+e_{22}U_x\right)$$

$$\frac{\partial s}{\partial y}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_y+e_{22}U_y\right)$$

$$\frac{\partial s}{\partial z}=\frac{\partial^2}{\partial t^2}\left(e_{12}u_z+e_{22}U_z\right) \quad (1)$$

where, $i,j>0(i,j=1,2,3....)$ are the incremental stress components; u,v,w are the displacement components in the solid along x,y,z directions respectively, whereas U,V,W

are the displacement components in the fluid present in the porous solid. Also, the angular componentsω_x, ω_y , ω_z are given by

$$\omega_x = \frac{1}{2}\left(\frac{\partial u_z}{\partial y} - \frac{\partial u_y}{\partial z}\right);\ \omega_y = \frac{1}{2}\left(\frac{\partial u_x}{\partial z} - \frac{\partial u_z}{\partial x}\right);\ \omega_z = \frac{1}{2}\left(\frac{\partial u_y}{\partial x} - \frac{\partial u_x}{\partial y}\right) \quad (2)$$

The mass coefficients e_{11} ,e_{12}, e_{22} are such that

$e_{11}>0$,$e_{12} > 0$, $e_{22} > 0$.

Stress, strain relation is a transversely isotropic poroelastic half space under initial P are

$$\sigma_{xx} = (A+2N+P)e_{xx} + (A+P)e_{yy} + (F+P)e_{zz} + M \in$$

$$\sigma_{yy} = A e_{xx} + (A+2N)e_{yy} + F e_{zz} + M \in$$

$$\sigma_{zz} = F e_{xx} + F e_{yy} + C e_{zz} + Q \in$$

$$\sigma_{xy} = N e_{xy},\quad \sigma_{yz} = L e_{yz},\quad \sigma_{zx} = L e_{zx}$$

$$S = M e_{xx} + M e_{yy} + Q e_{zz} + R \in \quad (3)$$

Strain components are expressed in terms of displacements

$$e_{xx} = \frac{\partial u_x}{\partial x};\quad e_{yy} = \frac{\partial u_y}{\partial y};\ e_{zz} = \frac{\partial u_z}{\partial z}.$$

$$e_{xy} = \frac{1}{2}\left(\frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}\right);$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_y}{\partial z} + \frac{\partial u_z}{\partial y}\right);$$

$$e_{yz} = \frac{1}{2}\left(\frac{\partial u_z}{\partial x} + \frac{\partial u_x}{\partial z}\right). \quad (4)$$

The displacements of love waves along *x*-axis and *z*-axis vanishes thus we have

$u=0$, $w=0$, $v= v\,(x, z, t)$

$U=0$, $W=0$, $V=V(x, z, t)$

Combining the equations (2)-(4) with the above displacements, equation (1) reduces to

$$\left(\frac{N-P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \frac{1}{2}\frac{\partial^2 v}{\partial z^2} = \frac{\partial^2}{\partial t^2}(e_{11} v + e_{12} V)$$

ISSN: 2366-1313

$$0 = \frac{\partial^2}{\partial t^2}(e_{12}v + e_{22}V) \quad (5)$$

Assuming harmonic wave solution in the form

$v_1 = f(z)\exp[i(kx+wt)]$

$V_1 = g(z)\exp[i(kx+wt)]$

where k is a wavenumber, equation (5) yields

$$\frac{L}{2}\frac{d^2 f}{dx^2} - k^2\left(\frac{N-P}{L}\right) = -w^2¿f + \rho_{12}g¿$$

$$0 = -w^2¿f + \rho_{22}g¿ \quad (6)$$

Using the solution of the above equations, the displacement v_1 and stress σ_{yz} in the layer can be obtained as

$$v_1 = ¿\, cos\gamma z + c_2 sin\gamma z ¿\exp[i(kx+wt)] \quad (7)$$

where $\gamma^2 = \frac{2❑^2}{L} ¿ - k^2\frac{(N-P)}{L}$

2. 2 Solution in the inhomogeneous half space

Under starting stress P´, the equations of motion in an inhomogeneous elastic solid (Biot, 1965) are as follows:

$$\frac{\partial\tau_{xx}}{\partial x} + \frac{\partial\tau_{xy}}{\partial y} + \frac{\partial\tau_{xz}}{\partial z} - P^{❑}\left[\frac{\partial\omega_x^{❑}}{\partial y} - \frac{\partial\omega_y^{❑}}{\partial z}\right] = \rho\frac{\partial^2 u}{\partial t^2}$$

$$\frac{\partial\tau_{yx}}{\partial x} + \frac{\partial\tau_{yy}}{\partial y} + \frac{\partial\tau_{yz}}{\partial z} - P^{❑}\left[\frac{\partial\omega_z^{❑}}{\partial x}\right] = \rho\frac{\partial^2 v}{\partial t^2}$$

$$\frac{\partial\tau_{zx}}{\partial x} + \frac{\partial\tau_{zy}}{\partial y} + \frac{\partial\tau_{zy}}{\partial y} - P^{❑}\left[\frac{\partial\omega_y^{❑}}{\partial x}\right] = \rho\frac{\partial^2 w}{\partial t^2}$$
(8)

Where $\tau_{xx}, \tau_{xy}, \ldots$ are incremental stressesu, v and w are displacement components

$$\omega_x^{❑} = \frac{1}{2}\left[\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}\right]$$

$$\omega_y^{❑} = \frac{1}{2}\left[\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\right]$$

$$\omega_z^{\square} = \frac{1}{2}\left[\frac{\partial u}{\partial x} - \frac{\partial u}{\partial y}\right]$$

For love waves $u=0, w=0$ and $v=v(x,z,t)$ we get

$$\frac{\partial \tau_{yx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} - ¿ \frac{P^{\square}}{2}\left[\frac{\partial^2 u}{\partial x^2}\right] = \rho \frac{\partial^2 u}{\partial t^2} \quad (9)$$

The non-zero stress-strain relations are

$$\square_{yx} = \mu\, e_{xy} = \frac{\mu}{2}\left[\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y}\right]$$

$$\square_{yx} = \mu\, e_{yz} = \frac{\mu}{2}\left[\frac{\partial w}{\partial x} + \frac{\partial v}{\partial z}\right] \quad (10)$$

In homogeneity of rigidity and density of the half-space are

$\mu = \mu^{\square}(1+\epsilon z)$, $\rho = \square^{\square}(1+z)$

Substituting μ in equation (10),

$$\tau_{yx} = \mu^{\square}\frac{(1+\in z)}{2}\frac{\partial v}{\partial x}$$

$$\tau_{yz} = \mu^{\square}\frac{(1+\in z)}{2}\frac{\partial v}{\partial z}$$

Using these stresses, the equation of motion (9) reduces to

$$\left(1 - \frac{P^{\square}}{\mu^{\square}(1+\in z)}\right)\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} + \left(\frac{\in}{1+\in z}\right)\frac{\partial v}{\partial z} = 2\frac{\square^{\square}}{\mu^{\square}}\frac{(1+z)}{(1+\epsilon z)}\frac{\partial^2 v}{\partial t^2} \quad (11)$$

Assuming wave solution in the form

v(z) = g(z) $e^{i(kx+wt)}$

and substituting it in equation (11), we get

$$\frac{d^2 g(z)}{dz^2} + \left(\frac{\in}{1+\in z}\right)\frac{dg}{dz} + \left(2\frac{\square^{\square}}{\mu^{\square}}\frac{(1+z)}{(1+\epsilon z)}c^2 - \left(1 - \frac{P^{\square}}{2\mu^{\square}(1+\in z)}\right)\right)k^2 g(z) = 0 \quad (12)$$

where $c^2 = \frac{\omega^2}{k^2}$

Taking g(z) = $\frac{\varnothing(z)}{\sqrt{1+\in z}}$ in above equation, we obtain

$$\frac{d^2 \varnothing(z)}{dz^2} + \left(\frac{1}{4} \frac{\in^2}{(1+\in Z)^2} - \left(\left(1 - \frac{P^{❑}}{\mu^{❑}(1+\epsilon z)} \right) - \frac{2c^2}{c_l^2} \frac{(1+z)}{(1+\epsilon z)} \right) k^2 \right) \varnothing(z) = 0$$

(13)

where $c_l^2 = \frac{\mu^{❑}}{❑^{❑}}$

Now defining the variables $\beta^2 = 1 - \frac{P^{❑}}{\mu^{❑}(1+\epsilon z)} - \frac{2c^2}{c_l^2} \frac{}{\in}$ and $s = \frac{2\beta k(1+\epsilon z)}{\epsilon}$,

equation (13) can be written as

$$\frac{d^2 \varnothing(s)}{ds^2} + \left(\frac{R}{2s} + \frac{1}{4s^2} - \frac{1}{4} \right) \varnothing(s) = 0 \quad (14)$$

where R = $2❑^2¿¿ = 2c^2¿¿$

The solution of equation (14) is

$$\varnothing(s) = E_1 W_{\frac{R}{2},0}(s) + E_2 W_{\frac{-R}{2},0}(-s) \quad (15)$$

where E_1, E_2 are constants and $W_{\frac{R}{2},0}(s)$ and $W_{\frac{-R}{2},0}(-s)$ are the Whittaker's functions. Since the solution is required in half-space, we must have $v(z) \to 0$ as $z \to \infty$ *i.e.* $\varnothing(s) \to 0$ as $s \to \infty$ and hence

$$\varnothing(s) = E_2 W_{\frac{-R}{2},0}(-s)$$

Now, the displacement component $v(z)$ can be written as

$$v_2(z) = E_2 ¿ \; e^{i(\omega t - kx)} \quad (16)$$

Considering up to linear terms of the Whittaker's functions, equation (16) can be written as

$$v_2(z) = E_1 e^{\frac{(1+\epsilon z)\beta k}{\epsilon}} \left(\frac{2\beta}{\in} \right)^{\frac{-R}{2}} (1+\in z)^{-\left(\frac{R+1}{2}\right)} \left(1 - \frac{\in \left(\frac{R+1}{2} \right)^2}{2\beta k(1+\epsilon z)} \right) \quad (17)$$

2. 3 Solution in the viscous liquid layer

The Navier Stokes equations of motion for viscous liquid are

$$\rho_l \left(\frac{\partial \mathbf{V}}{\partial t} \right) = -\nabla p + \frac{\eta_l}{3} \nabla(\nabla . \mathbf{V}) + \eta_l \nabla^2 \mathbf{V}, \quad (18)$$

where $\mathbf{V}(v_{xl}, v_{yl}, v_{zl})$ is the vector representing velocities in *x*, *y* and *z* directions,

ρ is density of liquid, η is coefficient of viscosity, *p* is over pressure and $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}$.

In the propagation of Love waves $u_{xl} = 0, u_{zl} = 0$, thus the velocity vector is $\mathbf{V} = (0, v_{yl}, 0)$ and that clearly gives $\nabla.\mathbf{V} = \mathbf{0}$. Hence, equation (20) of motion can be simplified to

$$\left(\frac{\partial v_{yl}}{\partial t}\right) = \frac{\eta_l}{\rho_l}\nabla^2 v_{yl}, \tag{19}$$

where η_l is the coefficient of viscosity and ρ_l is the density of the liquid.

Solving the equation (21), we can obtain solution as

$$v_{yl} = (D_1 i\omega\cos\alpha z + D_2 i\omega\sin\alpha z)\, e^{i(kx+\omega t)} \tag{20}$$

where $\alpha^2 = k^2 - \frac{i\omega\rho}{\eta}$.

Using the relation between stress and displacement in viscous fluid one can obtain the expression for stress as

$$S_{yz} = (-D_1 i\omega\alpha\eta_l \cos\alpha z + D_2 i\omega\alpha\eta_l \sin\alpha z)\, e^{i(kx+\omega t)} \tag{21}$$

3. Boundary conditions and frequency equation

The perfect bonding between the layers and half-space leads to the following boundary conditions:

at $z = h_1 + h_2$, $\sigma_{yz} = 0$

at $z = h_1$, $\sigma_{yz} = S_{yz}$

$v_1(z) = v_{yl}(z)$

at $z = 0$, $v_{yl}(z) = v_2(z)$

$$S_{yz} = ¿\,\tau_{yz} \tag{22}$$

Equation (22) represents five homogeneous equations in five constants. Hence, by eliminating these constants the frequency equation can be obtained as

$$\begin{vmatrix} \sin\gamma(h_1+h_2) & -\cos\gamma(h_1+h_2) & 0 & 0 & 0 \\ \cos(\gamma h_1) & \sin(\gamma h_1) & -i\omega\cos(\alpha h_1) & -i\omega\sin(\alpha h_1) & 0 \\ -\frac{\gamma L}{2}\sin(\gamma h_1) & \frac{\gamma L}{2}\cos(\gamma h_1) & i\omega\alpha\eta\sin(\alpha h_1) & -i\omega\alpha\eta\cos(\alpha h_1) & 0 \\ 0 & 0 & -i\omega & 0 & B \\ 0 & 0 & 0 & i\omega\alpha\eta & A \end{vmatrix} = 0 \quad (23)$$

where $A = \frac{\mu}{2}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}} e^{\frac{-\beta k}{\in}}\left[\frac{\in}{2}\left(\frac{R+1}{2}\right)^2\left(1+\frac{\in}{2\beta K}\left(\frac{R+1}{2}\right)+\frac{\in}{\beta K}\right)-\beta k-\frac{\in}{2}\left(\frac{R+1}{2}\right)\right]$

$B = e^{\frac{-\beta k}{\in}}\left(\frac{2\beta}{\epsilon}\right)^{\frac{-R}{2}}\left[\frac{\in}{2\beta K}\left(\frac{R+1}{2}\right)^2\right]$

$\gamma^2 = \frac{\square^2}{L}\left(\frac{N}{V_3^2}\right)-k^2\frac{(N-P)}{L}$, $\beta^2=1-\frac{P^{\square}}{\mu^{\square}(1+\epsilon z)}-\frac{2c^2}{c_l^2}\frac{}{\in}$ and

$$\alpha^2 = k^2 - \frac{i\omega\rho}{\eta}. \quad (24)$$

On further simplification the equation (23) reduces to

$$2\alpha\eta\left(A+B\alpha\eta\tan(\alpha h_1)\right)-\gamma L\left(A\tan(\alpha h_1)-B\alpha\eta\right)\tan(\gamma h_2)=0 \quad (25)$$

Case 3.1 In the limiting case $h_1 \to 0$, the frequency equation (25) reduces to

$$\tan(\gamma h_2) = \frac{-2A}{\gamma LB} \quad (26)$$

Equation (26) is the frequency equation of Loves waves at the interface of a transversely isotropic poroelastic layer and in-homogeneous half space.

Case 3.2 In the limiting case $h_2 \to 0$, the frequency equation (25) reduces to

$$\tan(\alpha h_1) = \frac{-A}{\eta\alpha B} \quad (27)$$

Equation (27) is the frequency equation of Loves waves at the interface of a Newtonian viscous liquid layer and in-homogeneous half space.

4. Numerical investigation

ISSN: 2366-1313

The effect of initial stresses and inhomogeneous parameters on the propagation of Love waves is discussed. In addition, the impact of viscosity coefficient and ratio of thickness of the two layers is also observed. The non-dimensional phase velocity vs. the non-dimensional wave number has been calculated and presented for various scenarios.

Figures 2 and 3 depict phase velocity for different values of non-dimensional in-homogeneous parameters $(ep = \varepsilon / k, si = \xi / k)$ taking constant non-dimensional initial stresses $P_1 = 0.5, P_3 = 1$ of the layer and the lower half-space respectively. In particular, Figure 2 is plotted for fixed $\xi / k = 0.1$ and different values of $\varepsilon / k = 0.3, 0.4, 0.5, 0.6$. It is observed that phase velocity decreases gradually with an increase in wave number. Phase velocity is more for higher values of the parameter ε / k. Fixed value $\varepsilon / k = 1$ and different values for $\xi / k = 0.1, 0.2, 0.3, 0.4$ are considered in figure 3. Phase velocity decreases with an increase in wave number but phase velocity is less for higher values of ξ / k. Also, it is noted that the differences in phase velocity for different values of ξ / k is little, particularly for higher values of wave number.

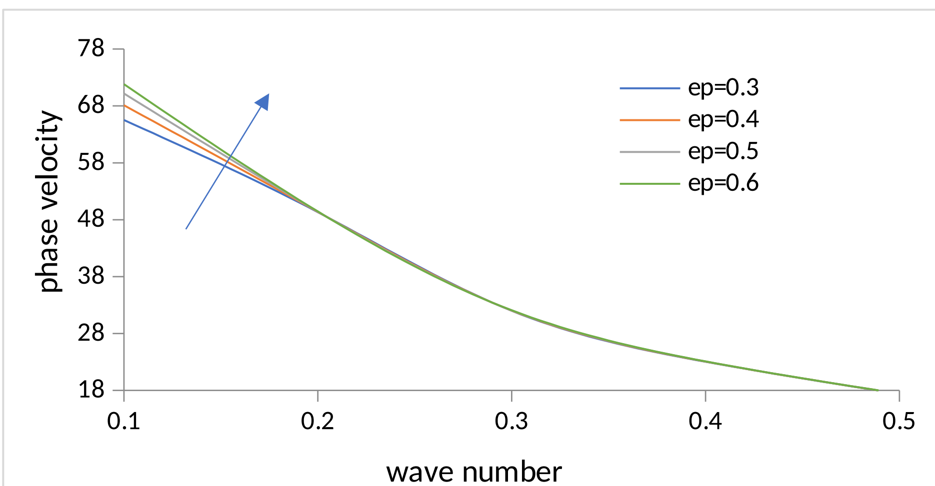

Fig. 2 Phase velocity in layer for different values of in-homogeneous parameters ε / k and constant value of $\xi / k = 0.1$.

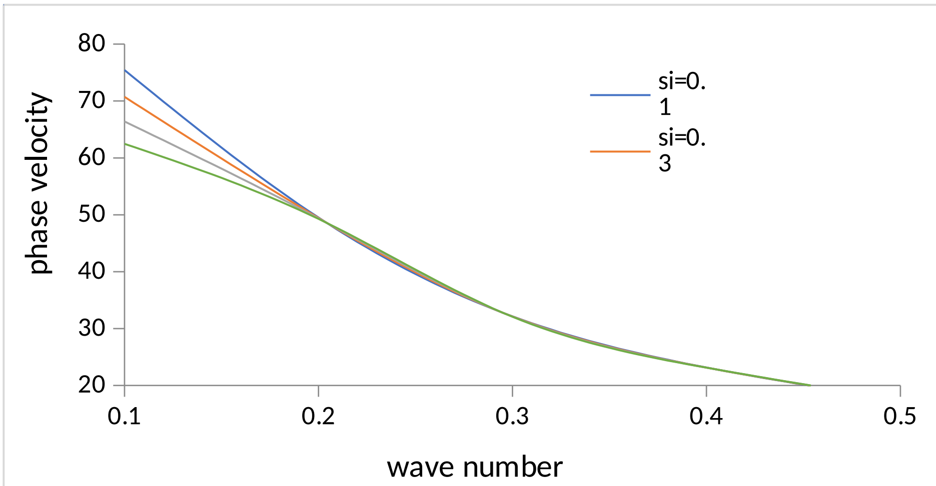

Fig. 3 Phase velocity in the layer for constant value of in-homogeneous parameters $\varepsilon / k = 1$ and different values of ξ / k.

Phase velocity in the transversely isotropic poroelastic layer against wave number for initial stress parameter $P_1 = 5$ of the layer and different values of initial stress parameter $P_2 = 0, 0.3, 0.6, 0.9, 1.2$ of lower half-space is presented in figure 4.

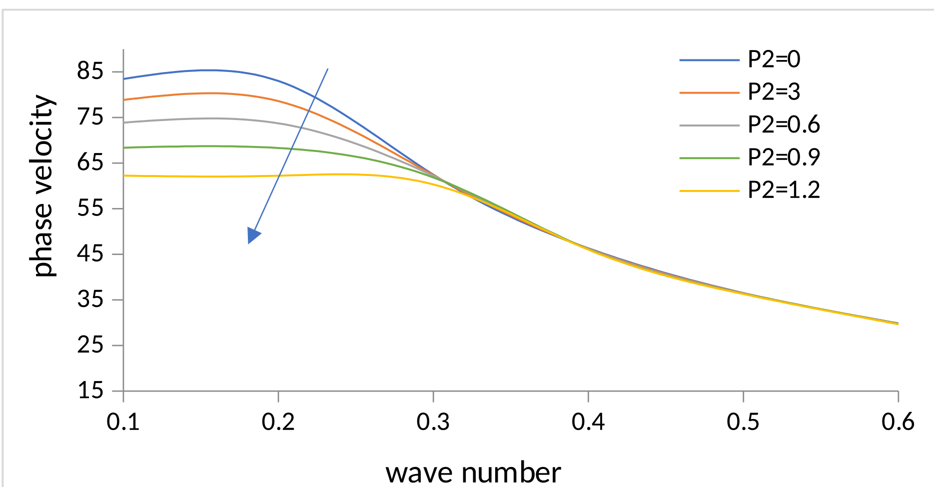

Fig. 4 Phase velocity in the layer for constant value of initial stress P_1=2 of layer and different values of initial stress P_2 of lower half-space

The non-dimensional in-homogeneous parameters are taken as $\varepsilon / k = 0.5, \ \xi / k = 0.1$. The phase velocity is low for higher values of initial stress P_2 of the lower half-space. Also, phase velocity is decreasing when wave number is increasing when initial stress P_2 of the lower half-space is taken as constant and different values of initial stress P_1 of the upper half-space is considered.

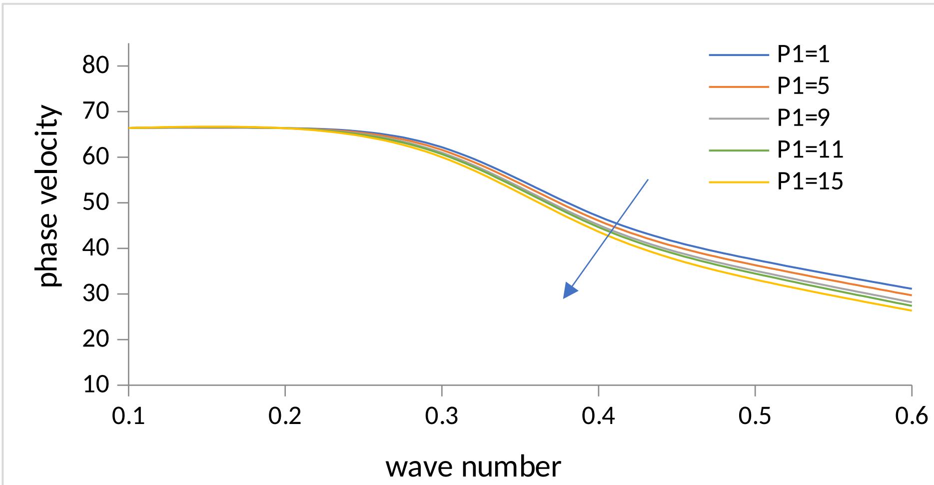

Fig. 5 Phase velocity in the layer for constant value of initial stress P_2 of the lower half-space and different values of initial stress P_1 of the layer

Figure 5 presents phase velocity for fixed initial stress parameter $P_2 = 0.5$ of the layer and different values of initial stress parameter $P_1 = 1, 3, 5, 7, 9$ of lower half-space. It is noted that the more the initial stress P_1 is taken the less phase velocity is obtained. The phase velocity is nearly same when wave number varies from 0.1 to 0.2. In figure 6, the behavior of phase velocity is presented for different values i.e. 0.1, 0.5, 1, 1.5 and 2 of viscosity coefficient. It is observed that phase velocity decreases as the wave number increases. Phase velocity increases as the viscosity coefficient increases.

Phase velocity for different values of the ratio h_2/h_1 of thickness of the solider layer to liquid layer is presented in Figure 7. In particular, the values of h_2/h_1 are 0.6, 0.7, 0.8, 0.9 and 1. It is noted that phase velocity decreases as wave number increases. Also, phase velocity is more for higher values of the ratio.

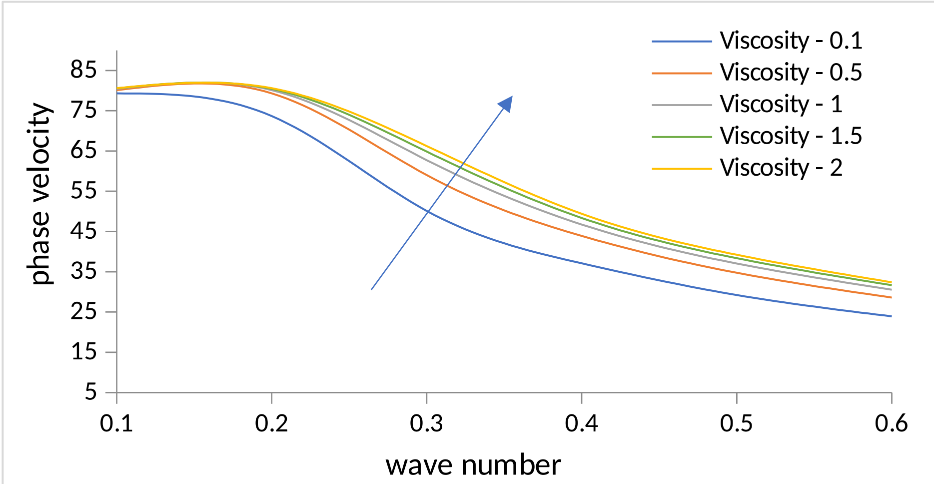

Fig. 6 Phase velocity in the layer for different values of viscosity coefficient

Phase velocity increases for wave numbers between 0.35 and 6, but decreases for wave numbers between 0.1 and 0.35.

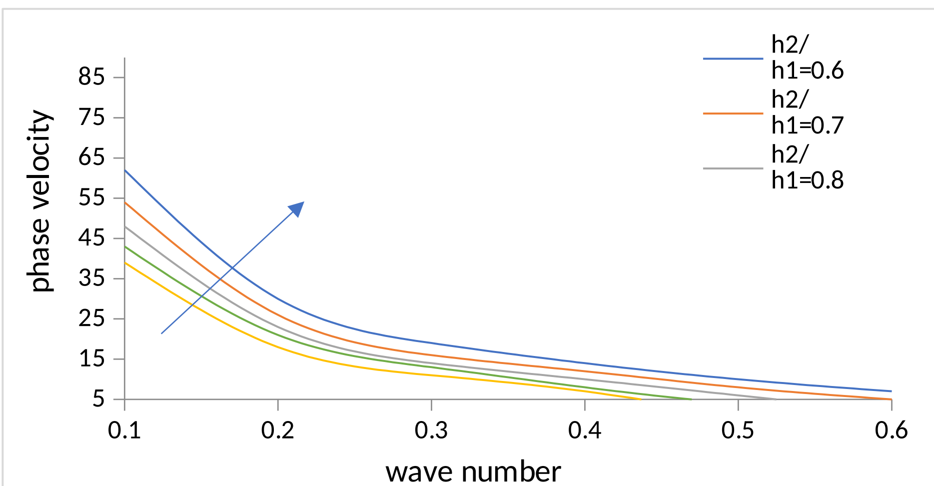

Fig. 7 Phase velocity for different values of the ratio h_2/h_1 of thickness of the two layers

5. Conclusion

Propagation of love waves at the interface between an inhomogeneous half-space and a transversely isotropic poroelastic layer is studied. The propagation of love waves is examined in relation to initial stresses and in-homogeneity characteristics. The study leads to the following results

1. Phase velocity of Love waves decreases with an increase in wavenumber.
2. Phase velocity increases with an increase in in-homogeneous parameter ε / k
3. Phase velocity drops as inhomogeneous parameter ξ / k increases. Thus, it is evident that phase velocity increases as the ratio ξ / ε of inhomogeneous parameters increases.
4. Phase velocity is less for higher values of initial stress of lower inhomogeneous elastic half-space.
5. An increase in an initial stress of lower half-space decreases the phase velocity.
6. Phase velocity increases as viscosity coefficient increases.
7. Phase velocity is more for higher values of the ratio of thickness of the solid layer to liquid layer.

References

1. Banghar A R, (1978) On propagation and attenuation of Love waves, Proc. Indian Acad. Sci., 88, 133-146.
2. Biot M A, (1956) The theory of propagation of elastic waves in fluid-saturated porous solid, J. Acous. Soc. Am., 28, 168-178.
3. Biot M A and Willis D G, (1957) The elastic co-efficients of the theory of consolidation, Journal of Applied Mechanics,vol. 24, 594-601.
4. Chattopadhyay A and De R K, (1983) Love type waves in a porous layer with irregular interface, International Journal of Engineering Science, 21, 1295-1303.
5. Deresiewicz H, (1962) The effect of boundaries on wave propagation in a liquid-filled porous solid-I, Bull. Seis. Soc. Am., 50, 599-607.
6. Deresiewicz H and Rice J T, (1962) The effect of boundaries on wave propagation in a liquid-filled porous solid-II, Bull. Seis. Soc. Am., 52, 595-626.
7. Gupta S, Chattopadhyay A and Majhi D K, (2010) Effect of initial stress on propagation of Love waves in an anisotropic porous layer, Journal of Solid Mechanics, 2, 50-62.
8. NageswaraNath C, Manoj Kumar J and Tajuddin M (2011) On the parametric model of loose bonding between two poroelastic half spaces, Journal of Vibration and Control, 18, 1261-1274.
9. NageswaraNath C, Manoj Kumar J and Ahmed Shah S, On propagation of Love waves in an infinite transversely isotropic poroelastic layer, Journal of Physics: Conference Series 662 (2015) 012004.

10. NageswaraNath C, Sravan Kumar V and Rajendar M, Propagation of Love waves in an isotropic poroelastic layer bounded between two viscous fluids, Materials Today: Proceedings 3 (2016) 4187-4192.
11. Poonam Khurana and Anil K Vashisth (2001) Wave propagation in a pre-stressed medium, Indian J. pure appl. Math., vol.32(8), 1201-1207.
12. Sharma, V., Goyal, R. & Kumar (2020) S. Love waves in a layer with void pores over a microstructural couple stress substrate with corrugated boundary surfaces. *J Braz. Soc. Mech. Sci. Eng.* **42**, 194.
13. Nageswaranath, Chinta., Ahmed, Shah, Syed., Ramesh, Modem., Venkata, Ramanamurthy, Mangipudi. (2019). A Study on Propagation of Waves in a Transversely Isotropic Poroelastic Layer Bounded between Two Viscous Liquids. Open Journal of Acoustics, 9, 1-12.
14. Bagno, A.M. (2020) Propagation of Waves in an Elastic Half-Space Interacting with a Viscous Liquid Layer. *Int Appl Mech* **56**, 700–707.