e-open sets and their Properties in Fuzzy Soft Topological Spaces

K. Kamala

Department of Mathematics, Pondicherry University,Kalapet-605107, Puducherry.
Email: krsaut@gmail.com

Abstract

This paper aims to initiate and explore the propertiesof fuzzy soft e-open sets fsδ-preopen fsδ-semiopenand introduce and investigate the properties of fuzzy soft e-interior(fs δ-preinterior, fsδ-semi interior) and fuzzy soft e-closure in fuzzy softtopological spaces. Moreover, we study the characterizations of fuzzy soft e-continuous and fuzzy soft e-quotientmappings in fuzzy soft topological spaces.

Keywords: Fuzzy soft e-open(closed), Fuzzy soft δ pre-open, Fuzzy soft δ semi-open Fuzzy soft e-interior(closure), Fuzzy soft e-continuous, Fuzzy softe-quotient map.

AMS Subject Classification:54A40,03E72.

1.Introduction

Zadeh[10] introduced the concepts of fuzzy sets. Soft sets theory was introduced by Molodtsov [3].The notion of a fuzzy soft set is investigated and discussed[4]. Chang[1] introducedand developed the concepts of fuzzy topology. In recent years, B. Tanay and M. B. Kandemir [5] much attention has been used to generalize the basic notions of fuzzy topology in a soft setting. In 2015, S. Hussain [6], introduced and explored the fuzzy soft semi-open(closed) sets by combining fuzzy soft sets and soft semi-open sets and investigated the properties of fuzzy soft semi-interior(closure) and fuzzy soft semi-open(closed) mapping. The concept of fuzzy soft pre-open and regular open sets was introduced by SabirHussain in 2016[7]. The concept of fuzzy e-open set wasintroduced by seenivasan.Vetal [9].In this paper,we introduce the notion of fuzzy soft e-open set, fuzzy soft δ-semi open, fuzzy soft δ-pre open, fuzzy soft e-continuousand studied their properties and their characterizationsof these functions are investigated.

2. PRELIMINARIES

Throughout the present paper, X , Y, Z denote the fuzzy soft topological spaces(fsts). Let f_A be a fuzzy soft set of X.The fuzzy soft closure (resp. fuzzy soft interior) of f_A is denoted by $\text{fs cl}(f_A)$(resp. $\text{fs int}(f_A)$ andA fuzzy soft set f_A is called fuzzy soft δ-open[8] if f_A=fs $cl_\delta(f_A)$ fuzzy softδ-closure(fuzzy soft δ interior) set f_A is denoted by $\text{fs cl}_\delta(f_A)$ ($\text{fs int}_\delta(f_A)$. A fuzzy soft set f_A is called fuzzy soft δ-open[8](resp. fuzzy soft semi-open, fuzzy soft pre open if f_A=fs $cl_\delta(f_A)$ (resp. $f_A \le \text{fs } cl(\text{fs int}(f_A))$ $f_A \le (\text{fs int}(\text{fs}cl(f_A))$) , the complement of fuzzy soft δ-open(resp. fuzzy soft semi-open, fuzzy soft pre open) set is called fuzzy softδ-closed(resp. fuzzy soft semi-closed, fuzzy soft pre closed.

3.FuzzySoft e-open,fsδ-preopen, fsδ-semiopen sets

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In this section we introduce fuzzy soft δ-semi open, fuzzy soft δ-pre open and fuzzy soft *e*-open sets in fuzzy soft topological spaces and study some of their properties

Definition 3.1. A fuzzy soft set f_A in a fuzzy soft topological space (X,τ,E) is called fuzzy soft δ-semi open (fsδ-semi open) (resp. fuzzy soft δ-pre open (fsδ-pre open) set iff $f_A \le \text{fs}cl(\text{fs}\operatorname{int}_\delta(f_A))$ (resp. $f_A \le \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$).
The complement of fuzzy soft δ-semi open(resp.fuzzy soft pre open) set is called fuzzy soft δ-semi-closed, fuzzy soft δ-pre closed set.
The union of all fsδ-semi open(resp.fuzzy soft δ-pre open) sets contained in a fuzzy soft set f_A in a fuzzy soft topological space X is called the fuzzy soft δ-semi interior(resp. fuzzy soft δ-pre interior) of f_A and it is denoted by $\text{fs}\operatorname{int}_{\delta s}(f_A)$ (resp. $\text{fs}\operatorname{int}_{\delta p}(f_A)$). The intersection of all fuzzy soft δ-semi closed(resp.fuzzy soft δ-pre closed)sets containing a fuzzy soft set f_A in a fuzzy soft topological space X is called the soft δ-semi closure (resp. fuzzy soft δ-pre closure) of f_A and it is denoted by $\text{fscl}_{\delta s}(f_A)$ (resp. $\text{fs}cl_{\delta p}(f_A)$)

Theorem 3.2 Let (X,τ,E) be an fsts and let f_A be a fuzzy soft set. Then the following hold.
(i) $\text{fs}\operatorname{int}_{\delta p}(f_A) = f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$
(ii) $\text{fs } cl_{\delta p}(f_A) = f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$
(iii) $\text{fs}\operatorname{int}_{\delta s}(f_A) = f_A \wedge \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$
(iv) $\text{fs } cl_{\delta s}(f_A) = f_A \vee \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$
Proof.(i) Since $\text{fs}\operatorname{int}_{\delta p}(f_A)$ is a fsδ preopen set , $\text{fs}\operatorname{int}_{\delta p}(f_A) \le \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$ and so $\text{fs}\operatorname{int}_{\delta p}(f_A) \le f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$. Conversely, let f_A is any fs openset and now $\text{fs}\operatorname{int}(\text{fs } cl_\delta(f_A) \le \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A)))) \le f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A)) \le \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A)$, since $f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A))$ is a fsδ-preopen set contained in f_A Therefore $f_A \wedge \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A)) \le \text{fs}\operatorname{int}_{\delta p}(f_A)$. Hence $\text{fs}\operatorname{int}_{\delta p}(f_A) = f_A \wedge \text{fs }\operatorname{int}(\text{fs}cl_\delta(f_A))$.
(ii) Now $\text{fs}\operatorname{int}_\delta(f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))) \le \text{fs}\,\text{cl}(\text{fs}\operatorname{int}_\delta(f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))))$
$\le \text{fs}\,\text{cl}(\text{fs}\operatorname{int}_\delta(f_A) \vee \text{fs}cl(\text{fs}\operatorname{int}_\delta(f_A)) = \text{fs}\,\text{cl}(\text{fs}\operatorname{int}_\delta(f_A) \le f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$, Hence by definition $f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$ is a fsδ-preclosed set which contains f_A Therefore, $\text{fs } cl_{\delta p} \le f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$. Conversely, since fs f_A is fsδ-preclosed, we have $\text{fscl}(\text{fs}\operatorname{int}_\delta(f_A) \le \text{fs}\,\text{cl}(\text{fs}\operatorname{int}_\delta(\text{fs}cl_{\delta p}(f_A))) \le \text{fs } cl_{\delta p}(f_A)$ and hence $f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$ $\le \text{fs}cl_{\delta p}(f_A)$. Therefore, $\text{fs } cl_{\delta p}(f_A) = f_A \vee \text{fs } cl(\text{fs}\operatorname{int}_\delta(f_A))$.
(iii) similar proof of (i)
(iv) similar proof of (ii)

Definition:3.3 A soft set f_A in a fsts (X,τ,E) is called(i) fuzzy soft *e*-open (fs*e*-open) set iff $f_A \le \text{fs}\operatorname{int}(\text{fs}cl_\delta(f_A)) \vee \text{fs}cl(\text{fs}\operatorname{int}_\delta(f_A))$ (ii)fuzzy soft *e*-closed (fs*e*-closed) set iff $f_A \ge \text{fs}cl(\text{fs}\operatorname{int}_\delta(f_A)) \vee \text{fs}\operatorname{int}(\text{fs}\,\text{cl}_\delta(f_A))$
Theorem:3.4 Let (X,τ,E) be fuzzy soft topological spaces,

(i) Any union of fuzzy soft e-open sets is a fuzzysoft e-open set, and
(ii) Any intersection of fuzzy soft e-closed sets is a fuzzy soft e-closed set.

Definition:3.5 In fsts X, a fuzzy soft set f_A is called the fuzzy soft e- interior if the union of all fs e-open sets contained in f_A and it is denoted by $\text{fs int}_e(f_A)$ and a fuzzy soft set f_A is called the fuzzy soft e-closure if the intersection of all fuzzy soft e-closed sets containing a fuzzy soft set f_A and it is denoted by $\text{fs cl}_e(f_A)$

Theorem:3.6 In fstsX, every fs open set and fs δ-preopen(fs δ-semiopen) set is fse-open set.
Proof: Let f_A be fs δ-preopen(fs δ-semiopen) set in X and $f_A \le \text{fs}\,\text{int}(\text{fs}\,cl_\delta(f_A))$ ($f_A \le \text{fs}\,cl(\text{fs}\,\text{int}_\delta(f_A))$)which implies $f_A \le \text{fs}\,\text{int}(\text{fs}\,cl_\delta(f_A)) \vee \text{fs}\,cl(\text{fs}\,\text{int}_\delta(f_A))$. Therefore f_A is fse-open set in X. but the converse is not true as shown in the following example.

Example:3.7 Let $X = \{a,b,c\}$, $E = \{e_1,e_2,e_3\}$ and $A=\{e_1,e_2\}$ $B=\{e_1,e_3\}$, $C=\{e_1\}$ and $D=\{e_2,e_3\}$ and let fuzzy soft sets

$f_A = \{f(e_1)=\{a_{0.4},b_{0.5},c_{0.6}\}, f(e_2)=\{a_{0.1},b_{0.2},c_{0.3}\}, f(e_3)=\{a_0,b_0,c_0\}\}$

$f_B = \{f(e_1)=\{a_{0.5},b_{0.6},c_{0.1}\}, f(e_2)=\{a_0,b_0,c_0\}, f(e_3)=\{a_{0.1},b_{0.4},c_{0.3}\}\}$,

$f_E = \{f(e_1)=\{a_{0.3},b_{0.4},c_{0.5}\}, f(e_2)=\{a_{0.4},b_{0.5},c_{0.1}\}, f(e_3)=\{a_{0.1},b_{0.6},c_{0.4}\}\}$

Consider the fuzzy soft topology $\tau=\{0_E,1_E,f_A,f_B,f_E,f_A\vee f_B,f_A\wedge f_B,f_A\vee f_E,$ $f_B\vee f_E,f_A\wedge f_E,f_B\wedge f_E,f_E\vee(f_A\wedge f_B),f_B\vee(f_E\wedge f_A),f_A\vee(f_E\wedge f_B),f_E\vee f_A\vee f_B\}$ defined over (X,τ,E). Now let us consider

$g_E = \{\text{g}(e_1)=\{a_{0.4},b_{0.6},c_{0.5}\}, \text{g}(e_2)=\{a_{0.5},b_{0.5},c_{0.2}\}, \text{g}(e_3)=\{a_{0.3},b_{0.6},c_{0.4}\}\}$ be a fuzzy soft sets. Then the fuzzy soft set g_E is fuzzy soft e open sets in (X,τ,E) but it is not fuzzy soft δ-pre open and also not fuzzy soft δ-semi open set in (X,E,τ).

Theorem:3.8 Let f_A be a fuzzy soft set in a fsts X, then,
$\text{fs}\,\text{int}_e(f_A) = \text{fs}\,\text{int}_{\delta p}(f_A) \vee \text{fs int}_{\delta s}(f_A)$.
Proof. It is obvious that, $\text{fs}\,\text{int}_e(f_A) \le \text{fs}\,\text{int}_{\delta p}(f_A) \vee \text{fs int}_{\delta s}(f_A)$
Conversely, from the definition we have,
$\text{fs}\,\text{int}_e(f_A) \le \text{fs}\,cl(\text{fs}\,\text{int}_\delta(\text{fs}\,\text{int}_e(f_A))) \vee \text{fs}\,\text{int}(\text{fs}\,cl_\delta(\text{fs}\,\text{int}_e(f_A)))$
$= \text{fs}\,cl(\text{fs}\,\text{int}_\delta(f_A)) \vee \text{fs}\,\text{int}(\text{fs}\,cl_\delta(f_A))$,
since $\text{fs}\,\text{int}_e(f_A)$ is fse-open set. By Theorem $\text{fs int}_{\delta p}(f_A) \vee \text{fs int}_{\delta s}(f_A) =$
$(f_A \wedge \text{fs}\,\text{int}(\text{fs}\,cl_\delta(f_A))) \vee (f_A \wedge \text{fs}\,cl(\text{fs}\,\text{int}_\delta(f_A))) = f_A \wedge (\text{fs}\,\text{int}(\text{fs}\,cl_\delta(f_A)) \vee \text{fs}\,cl(\text{fs}\,\text{int}_\delta(f_A)))$
$\ge f_A \wedge \text{fs}\,\text{int}_e(f_A) = \text{fs}\,\text{int}_e(f_A)$. Hence $\text{fs}\,\text{int}_e(f_A) = \text{fs}\,\text{int}_{\delta p}(f_A) \vee \text{fs int}_{\delta s}(f_A)$

Theorem:3.9 Let f_A be any fuzzy set infsts X, then
(i) $\text{fs}\ cl_e(f_A)^c = (\text{fs}\ int_e(f_A))^c$
(ii) $\text{fs}\,\text{int}_e(f_A)^c = (\text{fs}\ cl_e(f_A)))^c$.

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Theorem:3.10 Let f_A be a fuzz soft set of a fstsX. Then the following hold:

(i) f_A is fs δ-preopen if and only if $f_A \le \text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_{\delta p}(f_A))$

(ii) f_A is fse-open if and only if $f_A \le \text{fs}\ cl_{\delta p}(\text{fs}\,\text{int}_{\delta p}(f_A))$

(i) f_A is fs δ-semiopen if and only if $f_A \le \text{fs}\,\text{int}_{\delta s}(\text{fs}\,cl_{\delta s}(f_A))$

Proof :(i)Let f_A be fsδ-preopen. Then fs $\text{fs}\,\text{int}_{\delta p}(f_A) = f_A$ and $f_A \le \text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_{\delta p}(f_A))$.

Conversely, let $f_A \le \text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_{\delta p}(f_A))$. By Theorem3.2 we have,

$f_A \le \text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_{\delta}(f_A)) = \delta cl(F_A) \wedge \text{int}(\delta cl(F_A)) = \text{int}(\delta cl(F_A))$

Hence, f_A is fsδ-preopen.

(ii) Let f_A be fsδ-semiopen. Then fs $\text{fs}\,\text{int}_{\delta s}(f_A) = f_A$ and $f_A \le \text{fs int}_{\delta s}(\text{fs } cl_{\delta s}(f_A))$.

Conversely, let $f_A \le \text{fs int}_{\delta s}(\text{fs } cl_{\delta s}(f_A))$. By Theorem 3.2 we have,

$\le \text{fs int}_{\delta s}(\text{fs } cl_{\delta}(f_A)) = \text{fs } cl_{\delta}(f_A) \wedge \text{fs } cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\,cl_{\delta}(f_A))) \le \text{fs } cl(\text{fs}\,\text{int}_{\delta}(f_A))$

Hence, f_A is fsδ-semiopen.

(iii) Let f_A be fs$e$-open. Then $f_A \le \text{fs } cl(\text{fs}\,\text{int}_{\delta}(f_A)) \vee \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(f_A))$. By theorem 3.2, we have $f_A \le (\text{fs}\,cl(\text{fs}\,\text{int}_{\delta}(f_A)) \vee \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(f_A))) \wedge f_A$

$= (\text{fs}\,cl(\text{fs}\,\text{int}_{\delta}(f_A)) \wedge f_A) \vee (\text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(f_A)) \wedge f_A) \le \text{fs } cl(\text{fs}\,\text{int}_{\delta}(\text{fs int}_{\delta p}(f_A))) \vee \text{fs int}_{\delta p}(f_A)$

$= \text{fs cl}_{\delta p}(\text{fs int}_{\delta p}(f_A))$

Conversely, Let $f_A \le \text{fs cl}_{\delta p}(\text{fs int}_{\delta p}(f_A))$. By Theorem3.2, we have

$= \text{fs int}_{\delta p}(f_A) \vee \text{fs cl}(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))) \le \text{fs int}_{\delta p}(f_A) \vee \text{fs cl}(\text{fs}\,\text{int}_{\delta}(f_A))$

$= f_A \wedge \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(f_A)) \vee \text{fs cl}(\text{fs}\,\text{int}_{\delta}(f_A)) \le \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(f_A)) \vee \text{fs cl}(\text{fs}\,\text{int}_{\delta}(f_A))$

Hence, f_A is fs$e$-open.

Theorem:3.10 The following hold for a fuzzy soft set f_A of a space X:

(i) $\text{fs } cl_e(\text{fs}\,\text{int}_{\delta s}(f_A)) \le \text{fs}\,cl_{\delta p}(\text{fs}\,\text{int}_{\delta s}(f_A))$

(ii) $\text{fs}\,\text{int}_e(\text{fs}\,\text{cl}_{\delta s}(f_A)) \ge \text{fs}\,\text{int}_{\delta p}(\text{fs}\,\text{cl}_{\delta s}(f_A))$

(iii) $\text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_e(f_A) \le \text{fs int}_{\delta p}(\text{fs cl}_{\delta p}(f_A))$

(iv) $\text{fs } cl_{\delta p}(\text{fs}\,\text{int}_e(f_A) \ge \text{fs cl}_{\delta p}(\text{fs int}_{\delta p}(f_A))$

(v) $\text{fs}\,\text{int}_{\delta s}(\text{fs}\,cl_e(f_A) \le \text{fs}\,\text{cl}_e(f_A))$

(vi) $\text{fs}\,\text{int}_{\delta s}(\text{fs } cl_e(f_A) \ge \text{fs}\,\text{int}_e(f_A))$

Proof: (i) $\text{fs } cl_e(\text{fs}\,\text{int}_{\delta s}(f_A)) = \text{fs}\,\text{cl}_{\text{p}}(\text{fs}\,\text{int}_{\delta s}(f_A)) \wedge \text{fs}\,\text{cl}_{\delta s}(\text{fs int}_{\delta s}(f_A))$

$= \text{fs}\,\text{int}_{\delta s}(f_A) \vee \text{fs}\,\text{cl}(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta s}(f_A)) \wedge \text{fs}\,\text{int}_{\delta s}(f_A) \vee \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta s}(f_A))$

$= \text{fs}\,\text{int}_{\delta s}(f_A) \vee (\text{fs}\,\text{int}(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta s}(f_A))) \wedge \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta s}(f_A))))$

$\le \text{fs}\,\text{int}_{\delta s}(f_A) \vee \text{fs } cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta s}(f_A)))$

$\le \text{fs}\,cl_{\delta p}(\text{fs}\,\text{int}_{\delta s}(f_A))$

(ii) similar proof of (i)

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(iii) $\text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_e(f_A) = \text{fs}\,\text{int}_{\delta p}(\text{fs}\,cl_{\delta p}(f_A) \wedge \text{fs}\,cl_{\delta s}(f_A))$

$= \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\,\text{int}(\text{fs}\ cl_{\delta}(\text{fs}\ cl_{\delta p}(f_A))) \wedge \text{fs}\ cl_{\delta s}(f_A) \wedge \text{fs}\,\text{int}(\text{fs}\ cl_{\delta}(\text{fs}\ cl_{\delta s}(f_A)))$

$= \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\ cl_{\delta s}(f_A) \wedge \text{fs}\,\text{int}(\text{fs}\ cl_{\delta}(\text{fs}\ cl_{\delta p}(f_A)) \wedge (\text{fs}\ cl_{\delta s}(f_A)))$

$= \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\,\text{int}(\text{fs}\ cl_{\delta}(\text{fs}\ cl_{\delta p}(f_A)) \wedge (\text{fs}\ cl_{\delta s}(f_A)))$

$= \text{fs}\,\text{int}_{\delta p}(\text{fs}\ cl_{\delta p}(f_A)) \wedge \text{fs}\,\text{int}(\text{fs}\ cl_{\delta}(\text{fs}\ cl_{\delta s}(f_A)))$

$\leq \text{fs}\,\text{int}_{\delta p}(\text{fs}\ cl_{\delta p}(f_A))$

(iv) similar proof of (iii)

(v) $\text{fs}\,\text{int}_{\delta s}(\text{fs}\,cl_e(f_A) = \text{fs}\,\text{int}_{\delta s}(\text{fs}\,cl_{\delta p}(f_A) \wedge \text{fs}\,cl_{\delta s}(f_A))$

$= \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\ cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\ cl_{\delta p}(f_A))) \wedge \text{fs}\ cl_{\delta s}(f_A) \wedge \text{fs}\ cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\ cl_{\delta s}(f_A)))$

$= \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\ cl_{\delta s}(f_A) \wedge \text{fs}\ cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\ cl_{\delta p}(f_A)) \wedge (\text{fs}\ cl_{\delta s}(f_A)))$

$\leq \text{fs}\ cl_{\delta p}(f_A) \wedge \text{fs}\ cl_{\delta s}(f_A) = \text{fs}\ cl_e(f_A)$

(vi) similar proof of (v)

Theorem:3.11 Let f_A be a subset of a fsts X. Then the following hold:

(i) f_A isfs δ -preopen then $f_A \leq \text{fs}\ cl_e(\text{fs}\,\text{int}_{\delta p}(f_A))$

(ii) f_A isfs δ -semiopen and $f_A \leq \text{fs}\ \text{cl}_e(\text{fs}\ \text{int}_{\delta s}(f_A))$

(ii) f_A isfse-open then $f_A \leq \text{fs}\ cl_e(\text{fs}\,\text{int}_e(f_A))$

Proof :(i)Let f_A be fsδ-preopen. Then $\text{fs}\,\text{int}_{\delta p}(f_A) = f_A$

Consider, $f_A \leq \text{fs}\ cl_e(\text{fs}\,\text{int}_{\delta p}(f_A))$. By Theorem3.2 we have,

$= \text{fs}\ cl_{\delta p}(\text{fs}\,\text{int}_{\delta p}(f_A)) \wedge \text{fs}\ cl_{\delta s}(\text{fs}\,\text{int}_{\delta p}(f_A))$

$= (\text{fs}\,\text{int}_{\delta p}(f_A)) \vee \text{fs}\ cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))) \wedge (\text{fs}\,\text{int}_{\delta p}(f_A)) \vee \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A)))$

$= (\text{fs}\,\text{int}_{\delta p}(f_A)) \vee (\text{fs}\,cl(\text{fs}\,\text{int}_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))) \wedge \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))))$

$= (\text{fs}\,\text{int}_{\delta p}(f_A)) \vee (\text{fs}\,cl(\text{fs}\,\text{int}_{\delta p}(f_A)) \wedge \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))))$

$= (\text{fs}\,\text{int}_{\delta p}(f_A)) \vee \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A))) = \text{fs}\,\text{int}(\text{fs}\,cl_{\delta}(\text{fs}\,\text{int}_{\delta p}(f_A)))$ $\text{e} = \text{int}(\delta cl(F_A))$ Hence, f_A is fsδ-preopen.

(ii) similar proof of (i)

(iii) The proof follows from (i) and (ii)

4. Fuzzy soft e-continuous functions

Definition:4.1A mapping $\varphi : (X, \tau, E) \to (\text{Y}, \tau, E)$ is said to be a fuzzy soft *e*-continuous(fuzzy soft *e*-irresolute) if $\varphi^{-1}(f_A)$ is fuzzy soft *e*-open in τ_E for every fs open(fs e-open) set f_A in σ_E.

Theorem:4.2 For a function $\varphi : (X, \tau, E) \to (\text{Y}, \tau, E)$, the following are equivalent:

(a) φ isfs e-continuous.

(b) $\varphi^{-1}(f_A)$ isfs e-closed set in τ_E for each fuzzy soft closed set f_A in τ_E.

(c) $\text{fs}\ cl_e(\varphi^{-1}(f_A)) \leq \varphi^{-1}(\text{fs}\,cl(f_A))$ for every fs set f_A in σ_E.

(d) $(\varphi(\text{fs } cl_e\, g_B)) \le \text{fs } cl(\varphi(g_B))$ for every fs set g_B in σ_E
(e) fs $\text{int}_e(\text{fs cl}_e(\varphi^{-1}(f_A))) \le \varphi^{-1}(\text{fs}\, cl(f_A))$ for every fs set f_A in Y.
(f) $\varphi(\text{fs int}_e(\text{fs cl}_e(g_B))) \le \text{fs } cl(\varphi(g_B))$ for every fs set g_B in X.

Proof: $(a) \Rightarrow (b)$: Let f_A be a fs closed set of σ_E. Then $f_A{}^c$ is fs open set of Y. By Definition, $\varphi^{-1}(f_A{}^c)$ is fs e-open set in X. which implies $(\varphi^{-1}(f_A))^c$ is fs e-open set in X. Thus, $\varphi^{-1}(f_A)$ is afse-closed set of X.

$(b) \Rightarrow (c)$: Let f_A be any fs set of Y. Since fs $cl(f_A)$ is fs closed set in Y, then $\varphi^{-1}(\text{fs}\, cl(f_A))$ is fse-closed in X. Therefore fs $cl_e(\varphi^{-1}(f_A)) \le \text{fs } cl_e(\varphi^{-1}(\text{fs cl}(f_A))) = \varphi^{-1}(\text{fs}\, cl(f_A))$.

$(c) \Rightarrow (d)$: Let g_B be any fs set of X. By (c), $\varphi^{-1}(\text{fs}\, cl(\varphi(g_B))) \ge \text{fs}\, cl_e(\varphi^{-1}(\varphi(g_B)))$ $= \text{fs}\, cl_e(g_B)$. Thus $(\varphi(\text{fs}\, cl_e(g_B))) \le \text{fs } cl(\varphi(g_B))$.

$(d) \Rightarrow (e)$: Let f_A be any fs set of Y and $g_B = \varphi^{-1}(f_A)$. Consider $\text{fs}\, cl(f_A) \ge \varphi(\text{fs}\, cl_e(\varphi^{-1}(f_A)))$ which implies $\varphi^{-1}(\text{fs}\, cl(f_A)) \ge \text{fs}\, cl_e(\varphi^{-1}(f_A)) \ge \text{fs}\,\text{int}_e(\text{fs}\, cl_e(\varphi^{-1}(f_A)))$.

$(e) \Rightarrow (f)$: Let g_B be fs set in X and $f_A = \varphi(g_B)$, then $g_B = \varphi^{-1}(f_A)$. Consider $\text{fs}\,\text{int}_e(\text{fs}\, cl_e(g_B)) \le \text{fs}\,\text{int}_e(\text{fs}\, cl_e(\varphi^{-1}(f_A))) \le \varphi^{-1}(\text{fs}\, cl(f_A))$. Thus $\varphi(\text{fs int}_e(\text{fs cl}_e(g_B))) \le \text{fs } cl(\varphi(g_B))$.

$(f) \Rightarrow (a)$: Let f_A be fsclosed set of σ_E. Consider $\varphi(\text{fs int}_e(\text{fs cl}_e(g_B)))$ $\le \varphi(\text{fs int}_e(\text{fs cl}_e(\varphi^{-1}(f_A)))) \le \text{fs } cl(\varphi(\varphi^{-1}(f_A))) \le \text{fs } cl(f_A) = f_A$, which implies fs $\text{int}_e(\text{fs cl}_e(\varphi^{-1}(f_A))) \le \varphi^{-1}(f_A)$. Thus $\varphi^{-1}(f_A)$ is fs e-closed set in X. Hence φ isfs e-continuous.

Definition:4.3 Let (X, τ, E) and (Y, σ, E) be fuzzy soft topological spaces. A mapping $\varphi : (X, \tau, E) \to (\text{Y}, \sigma, E)$ is said to be afuzzy soft e-quotient(fs δ-pre quotient,fs δ-semi quotient) map if φ is onto map and $\varphi^{-1}(f_A)$ is fuzzy open inX then for any f_A is a fuzzy softe-open(fs δ-preopen , fs δ-semiopen) set in Y.

Definition:4.4 A mapping $\varphi : (X, \tau, E) \to (\text{Y}, \sigma, E)$ is said to be a fs e-open map if the image of every fs open set in τ_E is fs e-open set in σ_E.

Theorem:4.5 If $\varphi : (X, \tau, E) \to (\text{Y}, \sigma, E)$ is an onto map, fuzzy soft e-continuous and fuzzy e -open map, then f is a fuzzy soft e-quotient map.

Proof. Let f_A be any fs open set in Y. This implies $\phi^{-1}(f_A)$ is a fs e-open set in X since ϕ is a fs e-continuous and surjetive, fs e open map, $\varphi(\varphi^{-1}(f_A)) = f_A$ is fs e-open set in Y Hence φ is fuzzy soft e-quotient map.

Theorem:4.6 Let $\varphi : (X, \tau, E) \to (\text{Y}, \sigma, E)$ be an onto, fuzzy e open map and fse-irresolute map. Let $\phi : (Y, \sigma, E) \to (Z, \delta, E)$ be a fuzzy soft e-quotient map. Then $\phi \circ \varphi$ is afs e-quotient map.

Proof. Let f_A be any fse-open set in Z. Then $\phi^{-1}(f_A)$ is a fsopen set as ϕ is a fs e-quotient map. Since φ is fs e-irresolute, $\varphi^{-1}(\phi^{-1}(f_A)) = (\phi \circ \varphi)^{-1}(f_A)$ is a fs e-open set in X. Then $\varphi^{-1}(\phi^{-1}(f_A))$ is fuzzy e open in X. Since φ is fse openmapand surjective, $\varphi(\varphi^{-1}(\phi^{-1}(f_A))) = \phi^{-1}(f_A)$ is fs openset in Y. Since ϕ is a fuzzy soft e-quotientmap, f_A is a fuzzy soft e-open set in Z. Hence $\phi \circ \varphi$ is a fuzzy soft e-quotient map.

Theorem 2.4 If $\varphi : (X, \tau, E) \to (Y, \sigma, E)$ is an onto map, fs e -open map and fs δ -pre quotient(fs δ -semi quotient) then f is a fs e-quotient map.

Proof. Let f_A be any fs open set in Y. This implies $\phi^{-1}(f_A)$ is a fsfs δ -preopen(fs δ -semiopen) set in X since ϕ is a fs δ -pre quotient (fs δ -semi quotient)and This implies $\phi^{-1}(f_A)$ fs e-open set and φ is surjetive, fs e open map, $\varphi(\varphi^{-1}(f_A)) = f_A$ is fs e-open set in YHence φ is fs e-quotient map.

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