EIGEN FUZZY SETS OF FUZZY RELATION WITH APPLICATIONS

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1. INTRODUCTION

Abstract-When using max-min composition on fuzzy relations, it is quite doable to make use of Eigen fuzzy sets in order to arrive at an estimation of the highest and lowest possible values associated with the relevant variables. The largest eigen fuzzy set, which cannot be greater than it presently is, may be used to determine the maximum membership levels of any fuzzy set. This is because the greatest eigen fuzzy set cannot be bigger than it now is. However, the least eigen fuzzy set, which is the set with the lowest conceivable size, may be used to locate the minimum membership degrees in any fuzzy set. This is because it is the set with the fewest possible eigenvalues. Using eigen fuzzy set theory, it is possible to determine the least significant and most significant degrees of impacts or effects. The consequences that the findings of this research will have for future medical practice make it imperative that the study include evaluations of both the quality of medical care and the level of satisfaction experienced by patients. If one makes use of the tools that are made available by the eigen fuzzy set theory, one is able to get a highly accurate assessment of the success of a medical treatment as well as the degree to which a client is satisfied.

Key Words: fuzzy set theory, fuzzy relations, eigen fuzzy set.

The bulk of everyday problems stem from unclear thinking. To address this idea, we need some new notions since standard techniques from set theory and mathematics fall short. This idea is given a lot of weight in both fuzzy set theory and fuzzy logic. When asked to characterize fuzzy logic, its creator, Professor Lotfi A. Zadeh, uses the phrase "precise logic of imprecision."

Fuzzy sets are sets A and B where A is defined by a generalized characteristic function. "Classical sets" means "crisp sets" in the context of fuzzy set theory.

Fuzzy collection

The function A: X [0, 1] represents membership in the fuzzy set A. The degree of membership $y = A(x)$ [0, 1] for all elements x in X. Set of pairings $A = (x, y) = (x, A(x))$, x X (1.1), where X is a crisp universe set, totally determines the fuzzy set A.

The non-fuzzy set Supp(A) is a crucial component of the fuzzy set A.

If $A(x) > 0$, then Supp(A) = x X (= 1.2).

Fuzzy Connection

If the membership degree for each pair in the common set is known, then the two non-fuzzy sets are joined by the fuzzy relation.

• Definition
There are two definite Universes, X and Y. In a Fuzzy relation R X Y, each pair (x, y) is now associated with a membership degree R(x, y) [0, 1].(1.3)
µR : X × Y → [0, 1] (1.4)

Secondly, EIGEN FUZZY SETS

Inherent Fuzzy Sets

Let X be a finite set, and let R be a fuzzy relation between the members of X. (AoR = B, B X) where B is the maximum possible composition of R and A. It is said that A is an eigen fuzzy set associated with a particular relation R if and only if B = A.

If R is a fuzzy relation, then there must be an eigenfuzzyset A X, A: X [0, 1], A(x) [0, 1], x X that satisfies A0R = A, where R is a fuzzy relation determined by the membership function R : X X [0, 1], x, xj X.

The Biggest Eigen Fuzzy Set (GEFS)

If we have a finite example, we may utilize the Max-Min composition to learn about the largest eigen fuzzy set (GEFS) associated with R.
In the following paragraphs, we'll take a closer look at each of the three main formulae used to determine GEFS.

The Original Approach to Locating the GEFS

max(x, xJ), xJ = A1 (xJ), x = max(x, x). Let's refer to the fuzzy portion of X of interest as A1. The grades given to its members should sum up to the maximum allowed in each row of R.

While it is simple to prove that A0 is an eigen fuzzy set if we define it as a constant fuzzy set with the minimum of these

values, this does not necessarily make it the GEFS.

A series of fuzzy sets, denoted by (An)n, should be defined.

The limits of the decreasing sequence (An)n are (A0, A1)
The sequence is as follows: A0 .. An+1 An .. A3 A2 A1

Algorithm

R is a relation between X and Y, and R(x, xj) is a membership function for R.

1. Find the set A_1 identified by

$$\mu_A(x') = max_{x \in X} \mu_R(x, x'), \forall x' \in X$$

2. set the index $n=1$

3. calculate $A_{n+1} = A_n o R$

4. $A_{n+1} = A_n^{\rightarrow No \rightarrow n=n+1 \rightarrow go-to-step-3}_{\rightarrow Yes \rightarrow A_n = A_{n+1}}$

➢ **Example 2.1**
Let X={x1, x2, x3, x4} and R be given by following representation

$$R = \begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \end{array} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \left[\begin{array}{cccc} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{array}\right] \end{array}$$

The set A1 has the membership degree of xj found as the largest value in column j, j=1,2,3,4 and if thus determine as

$A_1 = 0.3 \ 0.8 \ 0.9 \ 0.6$
$A_0 = 0.3 \ 0.3 \ 0.3 \ 0.3$
$A_0 oR = A_0(Trivial - solution)$

For $n = 1$ we obtained

$$A_2 = A_1 oR = \begin{bmatrix} 0.3 & 0.8 & 0.9 & 0.6 \end{bmatrix} o \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 0.3 & 0.8 & 0.7 & 0.4 \end{bmatrix}$$

Since $A_2 \neq A_1$, we set $n=2$ in step 4, compose A_2 with R to get A_3

$$A_3 = A_2 oR = \begin{bmatrix} 0.3 & 0.8 & 0.7 & 0.4 \end{bmatrix} o \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 0.3 & 0.7 & 0.7 & 0.4 \end{bmatrix}$$

For $n=3$ we get

$$A_4 = A_3 oR = \begin{bmatrix} 0.3 & 0.7 & 0.7 & 0.4 \end{bmatrix}$$

A4 is accepted as greatest eigen fuzzy set (GEFS) of relation R as equality

$$A_3 = A_4$$

holds. We can observe that

$$A_4 \subseteq A_3 \subseteq A_2 \subseteq A_1$$

➢ **Second method of the GEFS determination**

There is no piece of music that necessitates this kind of evaluation, and there is no piece of music that receives such an evaluation. All that has to be done is to track down the variable whose value stays the same despite R's iterative reductions.

The constant features of this method are similar to those of the preceding strategy at every level.

Example 2.2:

$$R = \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

Steps:

Find the greatest element in each column of R, [0.3, 0.8, 0.9, 0.6]

The smallest of these element is denoted by r, in this case $r=0.3$ and the column containing is x_{1th} column.

Delete this 1st column and the same number row (1st) from R to get R^J

$$R' = \begin{bmatrix} 0.6 & 0.7 & 0.4 \\ 0.8 & 0.4 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$$

1. Set the value of r=0.3 in An(n is not known yet) at position of the deleted.

2. column

$$A_n = \begin{bmatrix} 0.3 & - & - & - \end{bmatrix}$$

3. Repeat all previous steps for R^J

$$R' = \begin{bmatrix} 0.6 & 0.7 & 0.4 \\ 0.8 & 0.4 & 0.1 \\ 0.5 & 0.3 & 0.2 \end{bmatrix}$$

When rj = 0.4 > 0.3 and if we obtain rj less than 0.3 from Rj, then we will accept the value of r at rj as the answer rather than the value of rj. Place the value 0.4 in the fourth place of the x-axis. An = [0.3 > 0.4]

$$R'' = \begin{bmatrix} 0.6 & 0.7 \\ 0.8 & 0.4 \end{bmatrix}$$

Where rjj = 0.7, which is higher than rj (rjj > rj), hence the value 0.7 may be found in the x3rd location.

An = [0.3 – 0.7 0.4]
At the conclusion, we were left with

Rjjj = [0.6]

And because rjjj=0.6 is a value that is smaller than rjj, we will substitute 0.7 for 0.6 in the x2th place of the equation for A4 = 0.3, 0.7, 0.7, and 0.4.

➤ **Third Method of the GEFS creation**

The assessment of at most successive powers of R using the max-min composition sense likewise results in GEFS. The method that is detailed below does provide a straightforward way to find GEFS, despite the fact that it is by no means simple.
Steps
Putting together A1 out of R

To get R2, just multiply R by itself to achieve the desired result.
A2 came into being since R2 was equal to RoR.
Simply adding R to R2 will give you R3.

In order to get A3 from R3, we need R3. If A is to be accepted, it has to be constructed in such a way that A+1 equals A. GEFs that are connected in some way to the letter R

➤ **Example 2.3:**

$$R = \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

From R we get $A_1 = [0.3, 0.8, 0.9, 0.6]$

composing R with R

$$R^2 = RoR = \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix} o \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

$$R^2 = \begin{bmatrix} 0.3 & 0.8 & 0.5 & 0.4 \\ 0.3 & 0.8 & 0.6 & 0.4 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

From R^2 we get $A_2 = [0.3, 0.8, 0.7, 0.4]$

$$R^3 = R^2 oR = \begin{bmatrix} 0.3 & 0.6 & 0.5 & 0.4 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.7 & 0.7 & 0.4 \\ 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

where $A_3 = [0.3, 0.7, 0.7, 0.4]$

$$R^4 = R^3 oR = \begin{bmatrix} 0.3 & 0.7 & 0.6 & 0.4 \\ 0.3 & 0.7 & 0.6 & 0.4 \\ 0.3 & 0.7 & 0.7 & 0.4 \\ 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

$$A_4 = A_3 = \begin{matrix} 0.3 & 0.7 & 0.7 \\ 0.4 \end{matrix}$$

➤ **Least Eigen Fuzzy set**

To determine which fuzzy set of a relation R has the least eigen value. There has been a little modification made to the GEFS algorithm.
➤ **Algorithm**

Let A relation $R \subseteq X \times X$ with membership function $\mu_R(x, x^J)$ is given

1. Find the set A_1 defined by

$$\mu_{A_1}(x') = min_{x \in X}(x, x'), \forall x' \in X$$

2. Set the index $n=1$

3. calculate $A_{n+1} = A_n oR$

4. $A_{n+1} = A_n \begin{matrix} \rightarrow No \rightarrow n=n+1 \rightarrow go-to-step3 \\ \rightarrow yes \rightarrow A_n = A_{n+1} \end{matrix}$

➤ **Example 2.4:**
Let continue with same relation R we used in previous examples

$$R = \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix}$$

We need to compute LEFS, thus we will choose the row in each column of R that has

the membership degree that is the lowest to get A1 = [0.1 0.5 0.3 0.1].

Composing A1 with R brings us to the solution for the case when n is equal to one.

$$A_2 = A_1 oR = \begin{bmatrix} 0.1 & 0.5 & 0.3 & 0.1 \end{bmatrix} o \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

According to the Max-Mix composition, the amount is found to be A2 (x1).

μA2 (x1) = max(min((0.1, 0.1), min(0.5, 0.3), min(0.3, 0.3), min(0.4, 0.2))) = max(0.1, 0.3, 0.3, 0.2)

= 0.3

Because A2 is equal to A1, we set n to 2, and then we composed A2 with R to create A3.

$$A_3 = A_2 oR = \begin{bmatrix} 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix} o \begin{bmatrix} 0.1 & 0.5 & 0.9 & 0.6 \\ 0.3 & 0.6 & 0.7 & 0.4 \\ 0.3 & 0.8 & 0.4 & 0.1 \\ 0.2 & 0.5 & 0.3 & 0.2 \end{bmatrix} = \begin{bmatrix} 0.3 & 0.5 & 0.5 & 0.4 \end{bmatrix}$$

That proves the equivalence A3 = A2, therefore that's OK. The relation R's set A3 is the one that has been approved as the LEFS (least eigen fuzzy set). It is clear that A1 comes before A2 and then A3

which validate the appropriate selection of the smallest collection
.

2. EIGEN FUZZY SETS WITH FUZZY NUMBERS

➢ **Minimum for two fuzzy Number**

• **Definition**

Let $N_1 = (m_{N_1}, \alpha_{N_1}, \beta_{N_1})$ and $N_2 = (m_{N_2}, \alpha_{N_2}, \beta_{N_2})$ are two fuzzy numbers, then

$$min(N_1, N_2) = (m_{N_1}, \alpha_{N_1}, \beta_{N_1})$$

If $m_{N_1} < m_{N_2}$ and

$$supp(N_1) \cap supp(N_2) = 0$$

or

$$min(N_1, N_2) = (min(m_{N_1}, m_{N_2}), max(\alpha_{N_1}, \alpha_{N_2}), min(\beta_{N_1}, \beta_{N_2}))$$

if $m_{N_1} \neq m_{N_2}$ or $m_{N_1} = m_{N_2}$ and

$$supp(N_1) \cap supp(N_2) \neq 0$$

(3.1&3.2)

➢ **Example 3.1:**

Let us take into consideration that N1 = (30, 2) 3 and N2 = (50, 1) 5 respectively. Both supp(N1) and supp(N2) equal the values [28, 33], and supp(N2) equals [49, 55]. have no components in common, therefore we can readily compare them and determine that the minimum value for N1 and N2 is the same as N1, as shown in figure 1.

$$min(N_1, N_2) = (m_{N_1}, \alpha_{N_1}, \beta_{N_1})$$

If $m_{N_1} < m_{N_2}$ and

$$supp(N_1) \cap supp(N_2) = 0$$

or

$$min(N_1, N_2) = (min(m_{N_1}, m_{N_2}), max(\alpha_{N_1}, \alpha_{N_2}), min(\beta_{N_1}, \beta_{N_2}))$$

if $m_{N_1} \neq m_{N_2}$ or $m_{N_1} = m_{N_2}$ and

$$supp(N_1) \cap supp(N_2) \neq 0$$

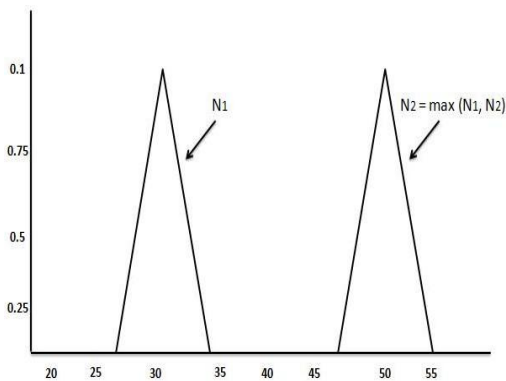so we decide min(N1, N2) = (min(30, 32), max(2, 1), min(33, 36)) = (30, 2, 33)
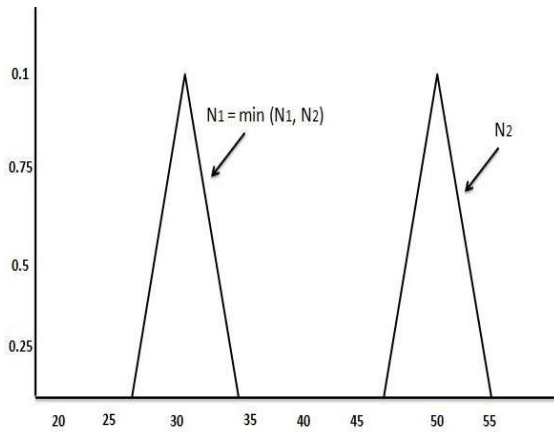
Figure 1: The minimum values for N1 are (30, 2, 3) and N2 are (50, 1, 5).

➢ **Example 3.2**

It can be shown that supp(N1) = [28, 33] and supp(N2) = [31, 36] for the cases where N1 = (30, 2, 3) and N2 = (32, 1, 4). The formula for this situation is as follows: (3.2).

$$supp(N_1) \cap supp(N_2) \neq 0$$

(3.3)

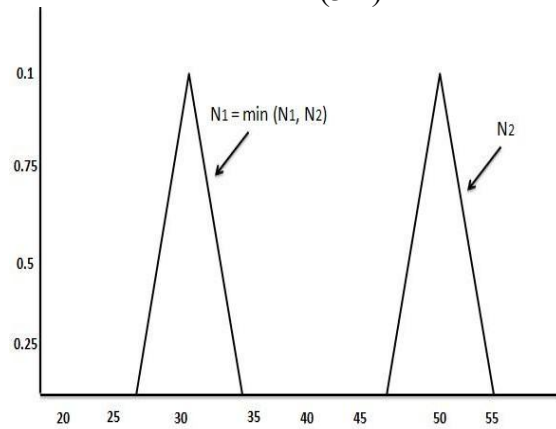**Figure 2: Minimum values for N1 are (30, 2) 3 and N2 are (50, 1) 5.**

➢ **Example 3.2**

It can be shown that supp(N1) = [28, 33] and supp(N2) = [31, 36] for the cases where

N1 = (30, 2, 3) and N2 = (32, 1, 4). The formula for this situation is as follows: (3.2).

Figure 3 : Minimum values for N1 are (30, 2) and N2 are (32-1).
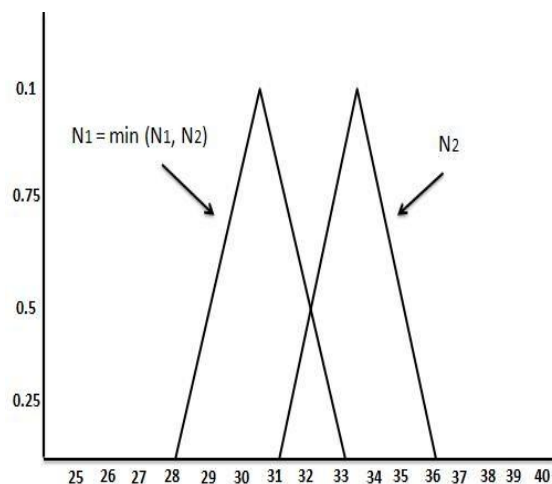
$$supp(N_1) \cap supp(N_2) \neq 0$$

(3.3)

The final output is seen in Figure 2. The same method may be used to find the larger of two fuzzy numbers (N1, N2) in a similar fashion.

➢ **Maximum for Two fuzzy numbers**
➢ **Definition**

Let $N_1 = (m_{N1}, \alpha_{N1}, \beta_{N1})$ and $N_2 = ((m_{N2}, \alpha_{N2}, \beta_{N2})$ are two fuzzy numbers, then Consider two fuzzy numbers,
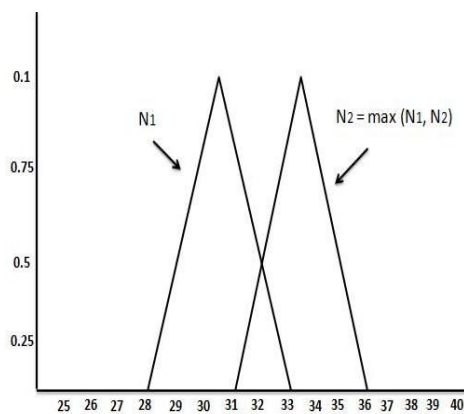
Figure 4: Maximum values for N1 = (30, 2, 3) and N2 = (32, 1, 4)

➢ **Example 3.4:**
Again we set $N_1 = (30, 2, 3)$ and $N_2 = (32, 1, 4)$ and applying formula 3.4 we get $max(N_1, N_2) = (32, 1, 4)$

Figure 5: Maximum values for N1 = (30, 2, 3) and N2 = (32, 1, 4)

3. APPLICATIONS OF EIGEN FUZZY SETS

We may make use of something that is known as an eigen fuzzy set in order to determine things such as the effectiveness of a therapy or the amount of satisfaction a consumer has with a product. The Eigen fuzzy set theory has the potential to be beneficial in a broad range of different scientific and technical settings and applications.

People who are happy and who make use of mobile gadgets provide one example.
By using eigen fuzzy set theory, we may be able to determine the highest and lowest levels of satisfaction experienced by mobile customers or users of a certain model. If it were the case, then this would be the result. For the purpose of establishing the veracity of this assertion, I enquired of 10 distinct individuals about their encounters with the Nokia Express Music 5800. On the list that was just provided, you may find traits such as the following:

• Being user-friendly and having a high usability is synonymous with F1 status.
Let's give this grouping of characteristics a name. F, where F2 indicates the level of sensitivity of the touch screen, and F3 indicates the amount of time the battery will survive.
F = {F1, F2, F3}
In response to the definition of a word that is shown below, one may establish a fuzzy connection in the following manner: "In a customer j, k = 1,..., n, the satisfaction of the jth feature is comparable to or stronger than the satisfaction of the kth feature." Using this information, one may make an approximate approximation of the highest and lowest possible values. When using this methodology, one is able to get a decent assessment of the extremes. A membership in Rmax(Fj,FK) refers to the level at which the jth and kth features of the statement defining Rmax are both true. This level is the one that is denoted by the notation; it is also the level that is referred to as a membership in Rmax(Fj,FK). In addition, this section outlines the criteria that must be met before some requirements are considered met. There is no break in continuity between levels 0 and 1 of membership status that may be earned in this organization.
The use of mathematical formulas

$$\mu_{R_{max}}(F_j, F_k) = \frac{b}{m}$$

where $j, k = 1, \ldots, n$

➢ **A Genetic Algorithm Based on Eigen Fuzzy Sets for Image Reconstruction**
The genetic algorithm (GA), which makes it possible to represent chromosomes, is

used to apply recombination operators to fundamental data structures. This makes it possible to simulate genetic variation. As a result of this, the strategy has the potential to act as a replacement for one of the potential solutions to a given issue. Even though they may be used for a wide range of tasks, genetic algorithms are most often thought of as optimization tools. This is despite the fact that they have a vast number of potential uses. To use the GA technique for image reconstruction, it is required to include GEFS and SEFS into the fitness function of the chromosome. Only then can a picture be reconstructed.

The locating of a solution code is considered to be the first step in the process of constructing a GA, since this is usually accepted as the standard practice. To put it another way, the answer that a chromosome is meant to represent must be encoded inside it in order for it to function properly. In the context of random pictures, a single pixel stands in for a single gene that is positioned along a chromosome. The allele value of a single gene is a positive integer that may range anywhere from 0 to 255, with the exact value dependent on where the gene sits within the range $X = 0$ to 255. An allele value is an attribute that can be inherited from a single copy of a gene.

In order to determine the fitness value inside a GA for the purpose of image reconstruction, we make use of two distinct eigen fuzzy set compositions. There are examples of the most extreme composition as well as the most extreme composition here. The GA accepts a wide variety of inputs, the most common of which are population sizes, generation counts, and a huge number of grayscale photos that are created at random. The picture that has been given the highest scores for fitness, GEFS, and SEFS is the one that is most likely to be chosen as a contender for the image that has been rebuilt.

CONCLUSION

The findings of this study as well as the survey provided insight into how the eigen fuzzy set theory might be used to data in order to ascertain the optimum degree of customer satisfaction. By using the concepts of eigen fuzzy sets and fuzzy connection, we were able to come at a number of conclusions on the most effective methods to increase the overall level of satisfaction that customers have with each product. The findings are shown in the following table.

It is possible that in the not too distant future, the idea of eigen fuzzy sets will be included in image enhancement algorithms in order to increase the amplification and overall quality of audio recordings, in addition to the quality of the picture that is viewed. In addition, comparable eigen fuzzy set processes may be utilized in lieu of traditional scientific research in order to improve the quality of any product based on statistical data obtained from a range of different sources. This may be done in order to increase the quality of any product. This might be done in place of research that would normally be carried out in a laboratory. This would serve as an alternative to the standard practice of carrying out research.

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