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A REIVEW ON DIFFERENTIAL EQUATION'S SOLUTION WITH TIME DILATION

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ABSTRACT

Differential and integral equations of integer and fractional order arise from mathematical modelling of issues in science and industry. Only under extremely special conditions and on extremely constrained surfaces are closed form solutions or precise answers for such problems achievable. Because of this, effective computational techniques for handling them have become increasingly crucial. Approximate solutions to differential and integral equations of integer and fractional order are computed in this thesis using a variety of numerical techniques. We hope to classify and evaluate the effectiveness of the chosen approaches. We address difficulties that scholars have encountered and stress the importance of multidisciplinary work in furthering the study of computational techniques for solving differential and integral equations. Fractional order differential equations of the Lane-Emden type are solved using a computational technique based on orthonormal Bernoulli's polynomials and their operational matrices

I. INTRODUCTION

Algebra is the bedrock of civilization. Real-world applications of numbers include counting, sketching, finding the company, and verifying the amount of money. Many other disciplines, including physics, chemistry, structural engineering, and computer science, rely on numbers for even the most basic calculations and calculations. Since the issues' evolving states can be either linear or non-linear, all processing apps describe them using either type of equation. Consequently, it's possible that a single answer won't work for both linear and non-linear issues.

These days, non-linear equations are the norm when it comes to solving

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problems in the actual world. Building precise answers to a nonlinear evolution equation is a crucial part of any thorough analysis of this type of equation. Finding precise answers to that equation is crucial to the study of complex physical systems.

Due to the intricacy and non-homogeneity of material characteristics, most real issues in science and engineering do not have a closed form answer. Therefore, estimated answers to such complicated issues must be obtained through the use of computational techniques. The finite difference method (FDM) was commonly used to address such issues in the 1960s. In contrast to FDM, the finite element technique (FEM) developed in the 1970s. While FEM divides the solution domain into several smaller sub-domains, FDM divides the solution domain into rows and columns of perpendicular lines to create grids. The sub-domains are depicted by triangles or quadrilaterals, and the unknown functions are estimated by the known functions, which are linear or higher order polynomials that rely on the physical coordinates used to determine the finite element form. After evaluating the finite element equations over each element, the results are compiled for the complete area. Many repeated techniques are used to answer the collection of algebraic equations derived in terms of the undetermined factors for the entire regions. There are many scientific and industrial areas where nonlinear wave events are observed.

- Fluid Mechanics
- Plasma Physics
- Optical Fiber
- Biology
- Solid State Physics
- Chemical Kinematics
- Chemical Physics
- Geochemistry and so on.

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The last few decades have shown that precise answers can aid in the discovery of previously unknown occurrences.

Non-linear equations are of interest because they accurately characterize the unique properties of many scientific and industrial uses. It is possible to get precise answers for non-linear problems in any area using homogeneous techniques. Simplified homogeneous balance methods, dimensional homogeneous balance methods, and etc. have all been suggested in previous study to address the problem of handling non-linear equations. However, they have a narrow field of usefulness. Design components for linear, non-linear, discontinuous, and random data produced by a wide range of engineering applications have recently become commonplace in the realm of computational applications. The uniform balance technique is widely used in all of science and engineering because of its ability to answer non-linear problems. Since there are three non-linear developing equations, it is recommended to use the uniform balance technique to solve them. So, the goal of this study was to use the uniform balance technique to solve some non-linear equations exactly. This study shows the use of the homogeneous balance technique for solving a number of non-linear equations, including Gardner, Burgers, and Fisher equations.

From the academic definition and description of numerical methods to their practical implementation as dependable and well-organized computer coders, numeric analysis investigates all facets of numerical solving of problems in continuous mathematics. When we say "continuous mathematics," we're referring to the branch of mathematics that studies things like real numbers, real-valued functions, etc. This type of issue features constantly varying factors and results from the practical application of math, geometry, and calculus. Natural sciences, social sciences, engineering, physics, building, medical, life science, economics, business management, and even the arts are not immune to

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these sorts of issues. Differential equations, systems of differential equations, integral equations, integra-differential equations, etc., arise from the mathematical modelling of these issues. Since closed form or precise solutions to such equations are only achievable under special conditions and for specific areas, a large part of numerical analysis studies is focused on finding approximations to the true answers. Therefore, in order to solve such mathematical issues, numerical analyzers and ap- plied scientists use a broad variety of instruments to create numerical techniques. When closed-form solution techniques cannot or will not work, numerical methods can be used to provide approximative but trustworthy and precise answers.

The proliferation of processing power and the sophistication of numerical analysis have allowed scientists and engineers to create increasingly accurate and detailed mathematical models in their fields. Using computers to apply numerical techniques for handling mathematical models of the actual world has increased the significance of computing. Mathematically inspired computer algorithms and numerical methods inform numerous fields of study, including but not limited to: machine learning and visualization; inverse issues; image processing; concurrent or dispersed processing; and symbolic or algebraic calculations.

II. ANALYSIS OF DELAY DIFFERENTIAL EQUATION

It was discovered a new way to use the continuous Runge-Kutta method to solve neutral delay differential equations (Enright et al., 1997). Current findings on instability for the first order delay differential equation and the first order neutral delay differential equation were spotted (Sung et al., 1999). For a similar polynomial equation with straightforward positive real roots, it was shown that the bifurcation analysis can be simplified if the delay term appears at the beginning of the equation (Jonathan Forde et al., 2004). With the help of step-by-step calculations and computational reversal of Laplace transforms, a novel approach was developed (Tamás Kalmár-Nagy, 2005).

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By estimating a harmonic function at the region's border, we were able to determine the overall stability of the delay differential equation (Leping, 2005). We have calculated the stability of the constant-delay trivial solution to the second-order linear delay differential equation (Ali Fuat Yeniçerioglu, 2007).

The inverse Laplace transform, in conjunction with the procedure for the linear delay-differential equation, can be used to locate a series of polynomial approximants to the transcendental function determining the delay equation's stability, which converges to the true value of the function (Tamas Kalmar-Nagy, 2009). Equations with linear and nonlinear components were solved numerically, analytically, and exactly using the Differential Transform Method (DTM) (Karakoç et al., 2009). For a wide variety of fractional delay systems, a useful computational method is provided for ensuring their stability (Farshad Merrikh-Bayat et al., 2009).

For linked systems of differential equations on networks, we verify the global-stability issue of equilibria (Michael et al., 2010). Stochastic delay differential equations instant stable area in terms of coefficient factors (Peng et al., 2011). For a stochastic delay differential equation, we investigate the impact of noise and delay from the perspective of mean-square stability (Peng et al., 2011). The use of stability switches has been presented as a means of gauging the robustness of complex-DDEs (Junyu et al., 2011).

Padé time delay estimate was established after solving and analyzing the issue of control design for incorporating unreliable time delay systems (Petr et al., 2011). Nonlinear disturbances and time-varying delays in a switched system were analyzed to determine their effect on the system's exponential stability. Furthermore, altered system configurations for resilient exponential stability in terms of linear matrix inequalities were achieved (Eakkapong et al., 2011).

Problems with the stability of linear continuous singular and discrete descriptor systems were elucidated, both in the limitless and limited time settings. Lyapunov stability and finite-time stability were both discussed at length. We studied the Lyapunov and non-Lyapunov stability characteristics of

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conjunctions between classes of linear continuous and discrete time delayed systems. These were offered by (Dragutin et al., 2011).

Second order delay differential equations with singular perturbations are solved using an exponential method (Awoke et al., 2012). For two-delay delayed systems, a mathematical technique was used to find a closed form for stability switching curves (Xihui et al., 2012). For fractional-order differential equations with a delay, we develop a numerical method using the Grunwald-Letnikov derivative (Zhen Wang et al., 2013). For a delay differential equation, we determined both the Ulam-Hyers stability and the extended Ulam-Hyers-Rassias stability (Diana et al., 2013).

With the Domain Decomposition Method (ADM), we show a computational solution to delay differential equations (DDE) (Ogunfiditimi, F.O, 2015). Solutions to a linear trinomial delay differential equation of fourth order were investigated for their oscillatory characteristics (Jozef et al., 2015). Furthermore, the oscillation principle for third-order functional differential equations with dampening was investigated (Martin et al., 2016). Conditions of global attractivity for nonlinear equations and exponential stability for linear equations with localized or designated delays have been met (Leonid et al., 2016).

The multi-pantograph delay differential equations with constant parameters are solved using the Sumudu Decomposition Method (SDM) (Sundas et al., 2016). Another technique for solving delay differential equations with constant latencies (delays) was given as well: the RK method, which is based on the Heronian mean (Vinci et al., 2016). All solutions to nonlinear neutral delay difference equations of first order with varying parameters were checked for their ability to oscillate (Murugesan et al.,2016). In-depth studies were conducted on the general solution and generalized Ulam-Hyers stability of an n-dimensional quartic functional problem (Murthy et al.,2016).

III. Integral Calculus

Since their introduction, functional equations have been of great interest to mathematics. Integral equations, in which the undetermined function is denoted by an integral symbol, have come to the fore in recent years. These sorts of formulae are ubiquitous in numerous scientific and mathematical disciplines. These integral equations are potent instruments that can be used to address a wide variety of real-world issues. All the starting and boundary value requirements for a differential equation can be represented by a single integral equation. In addition, the boundary value problem for a two-variable partial differential equation is transformed into a one-variable integral equation with an undetermined function. It is a major move forward to simplify a complex mathematical model into a single solution, and substituting differentiation with integration has additional benefits. The reality that integration is a levelling procedure has significant consequences for locating imprecise answers, which is where the advantages can be found. Whether seeking a precise or approximate answer, the construction of integral equations has proven to be helpful. Therefore, integral equations were widely studied and their theory was created thoroughly throughout the twentieth century.

Integral equation theory is a vital area of study in mathematics. The concept of an integral transform originated with the well-known Fourier integral formula. Transforming integrals effectively provides practical approaches to handling initial value and initial-boundary value issues for differential and integral equations, which is why they are so important.

History of mathematics, and more particularly the history of applied mathematics, is inextricably intertwined with the growth of the theory of integral equations. For the first time, the Norwegian scientist Niels Henrik Abel (1802-1829) provided a full answer to an integral equation for a tautochrone issue in mechanics [8]. The goal of this issue is to find

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the equation of a curve in the vertical plane such that the amount of time it takes for a mass point to move along this curve from a given positive height to the horizontal line is a known function of the height.

At the close of the 19th century, work started on a unified theory of integral equations. Several individuals, including V. Volterra (1896), E. Fredholm (1903), E. Schmidt (1907), and D. Hilbert (1911), have been given credit for developing this theory (1912). While Volterra did dabble in integral equations as early as 1884, he didn't really get going on his investigation until 1896. Readers were likely familiar with integral equations before 1888, when Du Bios-Reymond first proposed the word.

Potential theory unquestionably made the largest contribution to the advancement of integral equations. Integral equations were aided in their growth by mathematical physics models like dispersion in quantum mechanics, diffraction issues, ocean waves, and conformal mapping. Many more uses in science and industry can be understood through the use of integral formulae. Electrostatic, heat transfer, low frequency electromagnetic issues, acoustic and elastic wave transmission, electromagnetic dispersion problems, population development, particle transport problems, heat diffusion, etc. are all examples of applications where integral equations are commonly used. As a result, the results of different quantitative procedures are crucial in these fields.

IV. Mathematics of Fractions

The development of uses remains an important job, even though fractional calculus was presented over 300 years ago and has since been used in many fields of science and industry. Many scientific and technical fields, including those where nonlocality plays a vital part, continue to show and study models. Differentiable functions have characteristics that only specify integer order derivatives at a limited distance from the studied location. Therefore, non-locality in space and time cannot be characterized by the differential equation assessed for

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this position and including a limited number of integer-order derivatives. This nonlocality is a key motivator for the development of applications involving fractional calculus.

The so-called memory effect, in which prior events have an impact on the present, is evident in a wide variety of occurrences. The impedance of an electrical component, for instance, is proportional to the sum of the charges that have flowed through it over a given time interval. Attempting to describe and evaluate these memory effects using classical differential equations can be challenging. Since fractional derivatives are nonlocal, it is straightforward to include memory effects in systems that exhibit them. Therefore, fractional calculus has been shown to be one of the most efficient and potent mathematical instruments for explaining the memory and genetic characteristics of many processes and materials.

Certain natural events can be described more precisely than in traditional calculus by using fractional or non-integer order calculus. Aerodynamics, Fluid Mechanics [91], Earth System Dynamics, Viscoelasticity, Quantum Mechanics [70], Nuclear Physics, Control Theory, Biological Phenomena [92], Epidemic Processes [31], Signal and Image Processing, Rheology [162], Economics, Electrical Networks [22], Electromagnetic Theory [76], and Artificial Neural Networks have

Leibniz (1695), Liouville (1834), and Riemann (1892) were among the first to develop the fundamental concepts of fractional calculus; however, it was Abel (1823) who first used fractional calculus to solve an integral equation, which is a byproduct of the so-called tautochrone problem's formulation [8, 153]. Surely Liouville would have noticed Abel's answer if it hadn't been so remarkable, and he would never have made the first remarkable attempt to provide a rational meaning of the fractional derivative as he did. Since then, numerous well-known mathematicians have proposed various meanings of fractional derivatives; however, the

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most well-known and widely-used are those proposed by Riemann and Liouville, Grunwald and Plotnikov, and Caputo [102, 152]. Riemann-Liouville and Caputo both make use of the integral in their creation, with the former using a modified version of the Cauchy integral formula and the latter using the same basic idea.

The Cauchy algorithm for repetitive integration is stated in Section 1.3.1 as

$$\mathcal{J}^n f(t) = \frac{1}{(n-1)!}\int_0^t (t-s)^{n-1} f(s)ds, t > 0, n \in \mathbb{N}.$$

Definition 1.3.2. 152 Suppose that $t > 0, \alpha > 0, t, \alpha \in \Re$. Then the ReimannLiouville fractional integral of order α of a function $f(t)$ is defined as

$$\mathcal{J}^\alpha f(t) = \frac{1}{\Gamma(\alpha)}\int_0^t (t-s)^{\alpha-1} f(s)ds.$$

Some properties of Reimann-Liouville fractional integral operator $\mathcal{J}^\alpha$ are as follows:

1 For $\alpha, \beta \geq 0$

(a) $\mathcal{J}^\alpha \mathcal{J}^\beta f(t) = \mathcal{J}^{\alpha+\beta} f(t)$

(b) $\mathcal{J}^\alpha \mathcal{J}^\beta f(t) = \mathcal{J}^\beta \mathcal{J}^\alpha f(t)$

2 For $\gamma > -1, \mathcal{J}^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma+\alpha)} t^{\alpha+\gamma}$

Definition 1.3.3. Suppose that $\alpha > 0, t > 0$, and $t, \alpha \in \Re$. Then the Reimann Liouville fractional derivative of order $\alpha > 0$ of a function $f(t)$ is defined as

$$\begin{aligned} \mathcal{D}^\alpha f(t) &= \mathcal{D}^n \mathcal{J}^{n-\alpha} f(t) \\ &= \frac{d^n}{dt^n}\left[\frac{1}{\Gamma(n-\alpha)}\int_0^t (t-s)^{n-\alpha-1} f(s)ds\right], \end{aligned}$$

where $n-1 < \alpha \leq n$ and $n \in \mathbb{N}$.

Few main properties of Reimann-Liouville fractional derivative are as follows:

1 $\mathcal{D}^\alpha$ is left inverse to $\mathcal{J}^\alpha$, that is, $\mathcal{D}^\alpha \mathcal{J}^\alpha f(t) = f(t)$, but $\mathcal{J}^\alpha \mathcal{D}^\alpha f(t) \neq f(t)$

2 For $\gamma > -1, \mathcal{D}^\alpha t^\gamma = \frac{\Gamma(1+\gamma)}{\Gamma(1+\gamma-\alpha)} t^{\gamma-\alpha}$

3 For $\alpha \notin \mathbb{N}$ and $\gamma = 0$,

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$$\mathcal{D}^{\alpha}t^{0} = \mathcal{D}^{\alpha}1 = \frac{1}{\Gamma(1-\alpha)}t^{-\alpha},$$

the constant's Reimann-Liouville fractional derivative is not equivalent to zero. 1.3.4 Meaning [4%,46] The formula for the Caputo fractional derivative of a function f(t) of order is

$$\begin{aligned} {}^{\mathcal{C}}\mathcal{D}^{\alpha}f(t) &= \mathcal{J}^{n-\alpha}\mathcal{D}^{n}f(t) \\ &= \begin{cases} \dfrac{1}{\Gamma(n-\alpha)}\displaystyle\int_{0}^{t}(t-s)^{n-\alpha-1}\frac{d^{n}f(s)}{ds^{n}}ds, & n-1<\alpha<n, n\in\mathbb{N} \\ \dfrac{d^{n}f(t)}{dt^{n}}, & \alpha=n, n\in\mathbb{N} \end{cases} \end{aligned}$$

re $\alpha > 0$ and may take real or even complex values. This was first introduced by the Italian mathematician Caputo in 196%.

For the Caputo derivative, we have

$${}^{\mathcal{C}}\mathcal{D}^{\alpha}C = 0, \ (C \text{ is a constant }),$$

$${}^{\mathcal{C}}\mathcal{D}^{\alpha}t^{\gamma} = \begin{cases} 0, & \gamma \leq \alpha - 1, \\ \dfrac{\Gamma(\gamma+1)t^{\gamma-\alpha}}{\Gamma(\gamma-\alpha+1)}, & \gamma > \alpha - 1. \end{cases}$$

Similar to integer order differentiation Caputo derivative is linear.

$${}^{\mathcal{C}}\mathcal{D}^{\alpha}(\eta f(t) + \delta g(t)) = \eta\, {}^{\mathcal{C}}\mathcal{D}^{\alpha}f(t) + \delta\, {}^{\mathcal{C}}\mathcal{D}^{\alpha}g(t)$$

where η and δ are constants, and satisfies so called Leibnitz rule.

$${}^{\mathcal{C}}\mathcal{D}^{\alpha}(g(t)f(t)) = \sum_{k=0}^{\infty}\binom{\alpha}{k}g^{(k)}(t)\,{}^{\mathcal{C}}\mathcal{D}^{\alpha-k}f(t)$$

if $f(\tau)$ is continuous in $[0,t]$ and $g(\tau)$ has sufficient number of continuous derivatives in $[0,t]$.

CONCLUSION

The FEM was developed in response to the difficulty of structural analysis and elasticity issues in Civil and Aerospace Engineering. Initially, the FEM's value was only understood in terms of its ability to help with these types of applied engineering challenges. This led to the discovery of the driving force behind the creation of the FEM. Many other scholars quickly saw the method's potential in tackling nonstructural areas, thanks to the method's solid mathematical basis

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and adaptability. Over the past sixty years, advances in Science and Engineering have led to vastly superior FEM solutions for a wide range of boundary value issues. This resulted in the method's widespread adoption as a potent instrument for tackling a variety of difficult issues in Science and Engineering.

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